MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXII. JUNE 9, 1922. No. 8

Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Harold D. Babcock, Mount Wilson Observatory, Pasadena, California;
Bernard Benfield, Ph.B., 719 Rialto Building, San Francisco;
Captain Percy Francis Ward Blake, M.B.E., Beechworth, The Green, Wanstead, London, E. 11;
Clifford Charles Crump, B.S., M.A., Ph.D., Professor of Astronomy and Director of the Perkins Observatory, Ohio Wesleyan University, Delaware, Ohio;
Charles S. Cushing, 821 First National Bank Building, San Francisco;
Harcourt Leslie Dilks, Spring Bank, Broad Oak Park, Worsley, Manchester;
William Snyder Eichelberger, Ph.D., Director of the Nautical Almanac, Washington, D.C.;
Ferdinand Ellerman, Mount Wilson Observatory, Pasadena, California;
John Stratford Fleetcroft, 44 Middleton Square, London, E.C.;
Miss Caroline Furness, Ph.D., Professor of Astronomy, Vassar College, Poughkeepsie, New York;
Thomas Gray, Ashmead, Stockton Lane, York;
Miss Margaret Harwood, Director of the Maria Mitchell Observatory, Nantucket, Massachusetts;
John H. Hindle, Union Engineering Works, Haslingden, Lancs.;
James F. Hough, M.A., The School House, Brentwood;
John Richard Hudson, B.A., B.A.I., A.M.I.C.E.I., St. Ronan's, Clondalkin, Co. Dublin;
Francis P. Leavenworth, M.A., Professor of Astronomy in the University of Minnesota, Minneapolis, U.S.A.;
Frank Bowers Littell, Sc.D, Professor of Mathematics, U.S. Naval Observatory, Washington, D.C.;
Miss Frances Lowater, B.S., Ph.D., Assistant Professor of Physics, Wellesley College, Wellesley, Mass.;

Dr. Adriaan van Maanen, Mount Wilson Observatory, Pasadena, California;

Richard J. McDiarmid, Ph.D., Dominion Observatory, Ottawa, Canada;

Dr. Paul W. Merrill, Mount Wilson Observatory, Pasadena, California;

Willis Isbister Milham, Ph.D., Field Memorial Professor of Astronomy in Williams College, Williamstown, Mass.;

John A. Miller, Ph.D., Professor of Astronomy, Swarthmore College, Penn., and Director of the Sproul Observatory;

Daniel Walter Morehouse, B.S., M.S., Ph.D., Professor of Physics and Astronomy, Drake University, Des Moines, Iowa;

Jason John Nassau, Ph.D., Assistant Professor of Mathematics and Astronomy, Case School of Applied Science, Cleveland, Ohio;

Francis G. Pease, Mount Wilson Observatory, Pasadena, California;

Earl C. Slipher, Lowell Observatory, Flagstaff, Arizona;

Charles Campbell Smith, M.A., Dominion Observatory, Ottawa, Canada;

Thomas J. Smith, c/o John Browning, 37 Southampton Street, Strand, London, W.C. 2;

Frank Dickinson Urie, B.A., Assistant Director, Elgin Observatory, Elgin, Illinois;

C. I. Veale, Thomason College, Roorkee, India;

R. Stretton Webb, The Anchorage, Market Square, Paarl, S. Africa;

James Simms Wilson, c/o Troughton & Simms, Charlton, London, S.E. 7; and

Miss Anne Sewell Young, B.L., M.S., Ph.D., Professor of Astronomy and Director of Williston Observatory, Mount Holyoke College, South Hadley, Mass.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Miss Alice G. Bryant, A.B., M.D., F.A.C.S., The Austerfield, 502 Beacon Street, Boston, Mass., U.S.A. (proposed by A. E. Kennelly);

Charles Ashley Carus-Wilson, M.A., late Professor at McGill University, Montreal, Hanover Lodge, Lansdowne Road, London, W. 11 (proposed by W. W. Bryant);

Edward Crane, Devonshire House Hotel, Bournemouth (proposed by T. E. R. Phillips);

Henry William Evans, J.P., Harbour House, Solva, Pembrokeshire (proposed by C. W. Franklin);

Rev. Louis Alleyan Ewart, Ph.B., F.R.G.S., 13 Twyning Road, Edgbaston, Birmingham (proposed by A. J. Potts);

Luis C. Guérin, Observatorio Nacional, Córdoba, Argentina (proposed by C. D. Perrine);

Chester W. Hawkins, Observatorio Nacional, Córdoba, Argentina (proposed by C. D. Perrine);

Captain Alva E. Hernandez, Williammar, Broughton, Hants (proposed by D. A. Casey);

Miss Antonia C. de P. P. Maury, A.B., Hastings on Hudson, New York, U.S.A. (proposed by A. E. Kennelly);

Captain Arthur Routley Hutson Morrell, Commander of the British cable steamer *Henry Holmes* (proposed by D. A. Casey);

John A. Mowney, Melbourne Observatory, Victoria, Australia (proposed by C. J. Merfield);

Miss Cecilia Helena Payne, Newnham College, Cambridge, and 70 Lansdowne Road, London, W. 11 (proposed by L. J. Comrie);

Gordon Peace, Chief Chemist to the British India Corporation, Ltd., The Dale, Cawnpore, India (proposed by H. G. Tomkins);

Rikichi Sekiguchi, Assistant Director, Imperial Marine Observatory, Kōbe, Japan (proposed by H. F. Newall);

William George Steddy, 64 Northgate Street, Canterbury (proposed by E. W. Maunder);

Miss Lorna Mary Swain, Director of Studies and Lecturer in Mathematics, Newnham College, Cambridge (proposed by F. E. Cave-Browne-Cave);

Metford Watkins, B.A., Director of Mathematics, Fitzwilliam Hall, Cambridge, 86 Clifton Street, Swindon, Wilts (proposed by W. M. H. Greaves);

Robert Winter, Observatorio Nacional, Córdoba, Argentina (proposed by C. D. Perrine); and

M. L. Zimmer, Observatorio Nacional, Córdoba, Argentina (proposed by C. D. Perrine).

Fifty-one presents were announced as having been received since the last Meeting.

A Very Massive Star. By J. S. Plaskett.

Introduction.

The star B.D. 6° 1309, No. 2422 of the Harvard Revised Photometry and No. 47129 of the Henry Draper Catalogue, R.A. $6^h\ 32^m{\cdot}0$, Dec. $+6°\ 13'$ (1900), visual magnitude 6·06, is one of a number of early-type stars placed under spectroscopic observation on the completion of the first radial-velocity programme about ten months ago. This restricted programme, whose observation and discussion were undertaken by myself, consisted of all the known Oe and Oe5 stars within reach at this observatory, 36 in number north of declination $-25°$, and 13 other stars, suspected of being analogous in spectral type, which were selected by the kind assistance of Miss Cannon from the Henry Draper Catalogue.

The principal purpose in mind in entering upon the investigation of

these stars, in addition to careful analyses of their spectral characteristics and peculiarities and determination of their radial velocities, was to obtain so far as possible their masses. Many of them had previously been shown to be spectroscopic binaries, and three orbits already determined had shown exceptionally large mass-functions. Ludendorff, in his exhaustive analysis of spectroscopic binary systems,* had shown there was a progressive increase of mass with increase of temperature, with change of spectral type, culminating in the three very massive systems of type Oe and Oe5. According to Campbell and Moore,† the masses of several of the planetaries are relatively high, N.G.C. 7009, 162 times the sun, while the analogy between the planetaries and Wolf-Rayet or O-type stars has been clearly shown by Wright.‡ It was, therefore, felt to be useful to closely examine these spectral classes, which must from their limited numbers as compared with the majority of the spectral classes be exceptional in some characteristics, one of the most probable being, as already indicated, the mass. Although the investigation of these stars is not sufficiently far advanced to make any definite statements about their general characteristics, it has been deemed desirable to publish the information already obtained about one of their number, the star B.D. 6° 1309, which has proved to be about four times as massive as any star previously investigated.

The Spectrum.

Its spectrum is classed as A5 in Harvard 50, but this is amended in the Henry Draper Catalogue, where it is called Bop. In the appended remarks, which led to its inclusion in the programme, it is stated : "H_β is not seen as a distinct dark line, and appears to be slightly bright. The lines are narrow. Line 4200·3 is strong."

The first spectrogram obtained on 1921 December 16 showed doubled spectrum lines of type about Oe5. As the displacement was large, special attention was devoted to it, and spectrograms obtained as frequently as possible. Thirty have been obtained and measured to determine the spectroscopic orbit. Although both spectra appear and can be measured, that of the principal component is much more intense and sharp, and the description applies principally to it. The spectrum is somewhat peculiar, for, though having basically the absorption lines with relative intensities characteristic of Oe5 spectra, emission is certainly present, but not as definitely so as in spectra of Class Oe, and this spectrum is probably midway between Oe and Oe5. Broad emission bands about 25 A. wide, with strong central absorption, appear in the positions of H_β, H_γ, and H_δ. Emission can also be seen at the enhanced helium "Pickering" lines at 4542 and 4200, and is visible on one or two plates near the helium absorption 4471. Perhaps the most characteristic feature of the spectrum is the complete disappearance of absorption at the enhanced helium line 4686 and its replacement by broad emission with central decrease of intensity, perhaps better

* *A.N.*, 5046.
† *Lick Obs. Pub.*, 13, 177.
‡ *Ibid.*, p. 248 *et seq.*

described as double slight emission on each side of the position of 4686. Emission is, however, by no means a prominent feature of the spectrum, and can only be distinctly seen on a few of the spectra, requiring apparently certain rather narrow limits of exposure and development to bring it out to best advantage. Were it not for the absence of 4686 absorption, it would not be suspected on most of the plates.

A list of all the lines measured in the spectra of 6° 1309 is given in the following table, with their identifications and approximate intensities. A few other faint and diffuse lines were occasionally glimpsed, for example, oxygen lines around 4070, but were not sufficiently defined to be measured. Those lines to which an asterisk is appended were measured on the secondary spectrum as well as on the primary.

Lines in Brighter Component 6° 1309.

Wave-Length. I.A.	Identification.	Intensity.	Wave-Length. I.A.	Identification.	Intensity.
3933·68	Ca	3	4340·467*	H_γ	5
3964·727	He	1	4379·09	N	1
3968·48	Ca	2	4387·928	He	2
3970·075	H_ϵ	2	4471·477*	He	4
4009·27	He	1	4481·195	Mg	1
4026·189*	He	3	4510·91	N	2
4088·90	Si	2	4514·865	N	2
4097·327	N	3	4523·59	N	1
4101·738	H_δ	4	4541·61*	He	3
4116·16	Si	2	4685·80	He	1
4120·812	He	1	4713·143*	He	2
4143·759*	He	1	4861·326*	H_β	4
4200·06*	He	2	4921·929*	He	2

The calcium lines H and K are strong, narrow, and beautifully defined. They appear to be absolutely stationary in the star, and do not partake of the large excursions of the other lines. The lines of enhanced nitrogen 4097, 4511, 4515, 4524, though not very strong, are fairly well defined and give reliable measures. The enhanced nitrogen line 4379 is very faint, and can only be seen on three or four plates. The helium lines 4026, 4471, 4713, 4922 are all strong lines, but though rather diffuse appear to give fairly reliable measures. The Pickering lines of helium 4542 and 4200 are fairly strong and well defined, although their intensity is perhaps slightly decreased by emission, which can be seen beside them on some plates. H_γ is the only hydrogen line which gave reasonably good velocity measures, and even these were sometimes discrepant, while H_β and H_δ, the latter being very broad and complex, could not be used. The hydrogen lines are too diffuse to be able to recognise the presence of the enhanced helium components,* but the uncertainty of the measures indicates the action of some disturbing blend, and they are probably present. As previously stated, enhanced helium 4686 cannot be measured as absorption, but a partially reversed

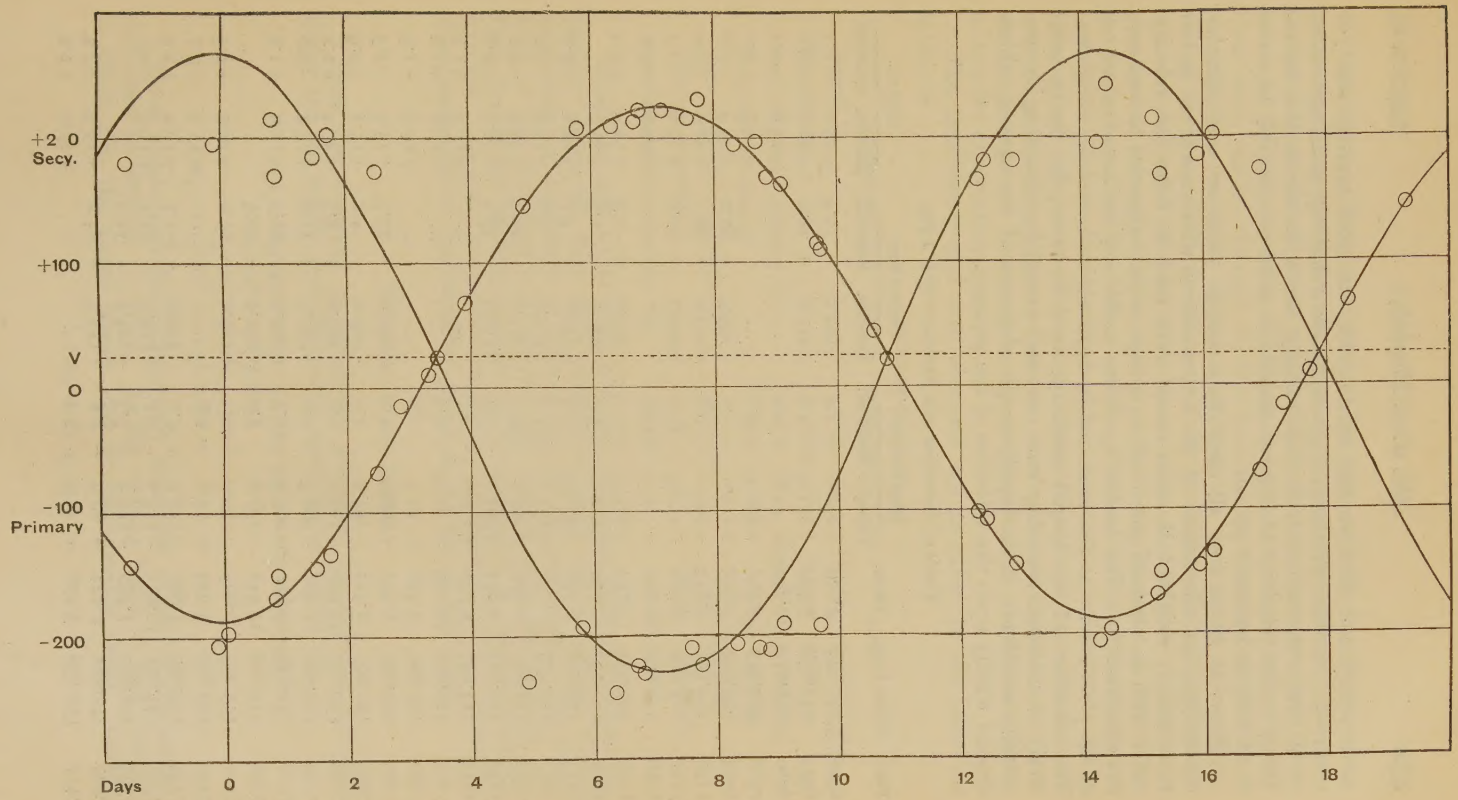
* H. H. Plaskett, *Nature*, 108, 209, 1921 ; and *Pub. D.A.O.*, 1, No. 30.

weak emission is undoubtedly present. Although the lines are not well defined they are much sharper than on many of the Oe5 stars, and on such eclipsing variable B's as Y Cygni, which has a combined mass 32 times the sun, but in which the probable error of measurement of a single plate is over three times as great as for 6° 1309. Although unlike any other Oe or Oe5 spectrum obtained, there are no striking peculiarities to indicate exceptional characteristics.

The spectrum of the second component is very weak, and the measures very difficult and uncertain. The Pickering helium lines 4542 and 4200 seem relatively the strongest in this spectrum, indicating possibly a slightly earlier type, but the stronger helium lines 4026, 4472, 4713, 4922 also appear and have been measured. H_β and H_γ in the second spectrum have generally been measured, but were only given low weight on account of faintness and breadth. Although H_δ is probably also present, the combined spectrum is so complex and diffuse in this region as to prevent any measures of either principal or secondary being made. There are no traces of companions to the enhanced nitrogen lines which appear in the primary spectrum, and they are probably too weak to show. From the relative intensities of the lines of the two spectra, one would judge there must be nearly a magnitude difference in brightness, but the relative masses differing by only 20 per cent. seem to contradict this estimate. The computed absolute brightness is only 0·2 magnitude fainter than the primary, so that the relative weakness of the lines must be due to their relative faintness in the secondary spectrum, the contrast being insufficient to clearly show them against the strong combined continuous spectrum of the two stars.

The Spectroscopic Orbit.

For the determination of the spectroscopic orbit 30 spectrograms were secured between 1921 December 16 and 1922 April 5. They were all of single-prism dispersion, 27 being obtained with the medium-focus camera giving 29 A. to the millimetre at H_γ, and 3 with a short-focus camera giving a dispersion of 55 A. After an interval of about two months, a period of about fourteen days appeared to best satisfy the observations; but as this period with the large range, about 400 km. of the primary and 500 of the secondary, would entail a mass of over 100 times the sun, the whole series was reviewed to see if a shorter period would not satisfy the observations. It is sometimes possible to fit long-period observations, especially if always obtained about the same hour-angle, into a period only slightly greater than one day, and all such possible periods were tried without success. In order to make certain that the period was not one or two days or shorter, two spectrograms were obtained with as long an interval as possible between exposures, about four hours, on four separate nights. If the star had a shorter period than fourteen days, such as one or two days, these observations would clearly show this by their large difference in velocity. But in each case the difference was only what should be expected from a fourteen-day period, and there remained no doubt this was the approxi-



Velocity Curve of B.D. 6° 1309.

mate period and that we had to do with the most massive star yet observed. Further evidence is given by the relatively good agreement of all the observations with the velocity curve shown in the figure. Thirty radial velocities of so great amplitude could not possibly be made to fit with an incorrect period.

These 30 plates were all carefully measured twice on a micrometer microscope, and the means of the two measures which are given in the table below, weighted in some cases, were used to form the velocity curve with an assumed period of 14·4 days, which appeared to best suit the observations. The second spectrum could only be measured on 21 of the plates, and was also measured in duplicate and carefully weighted. The measures of the second spectrum were, however, felt to be relatively so uncertain that they were not used, except for obtaining its own velocity amplitude, in the spectroscopic elements of the system, which depend wholly upon the velocities of the primary.

Table of Velocities and Residuals of 6° 1309.

Plate.	Julian Date.	Phase.	Brighter Star.		Fainter Star.		Calcium.	
			Velocity.	Residual.	Velocity.	Residual.	Velocity.	Residual.
6906	3,040·949	9·079	+162·8	+ 2·4	−192·0	− 35·4	+18·1	+2·1
6953	053·838	7·554	+215·3	− 4·6	−211·8	+ 16·0	+16·6	+0·6
6960	056·874	10·590	+ 44·6	+ 0·9	...	...	+11·6	−4·4
7006	067·849	7·151	+222·4	− 0·7	...	...	+14·0	−2·0
7095	084·802	9·695	+114·6	− 5·1	−195·3	− 87·2	+15·4	−0·6
7138	104·753	0·813	−169·1	− 0·8	+215·0	− 26·1	+18·0	+2·0
7142	105·660	1·720	−132·5	−10·4	+202·1	+ 20·9	+13·8	−2·2
7168	106·802	2·862	− 14·0	+14·7	...	...	+20·6	+4·6
7193	109·733	5·793	+207·5	+14·1	−193·6	+ 2·3	+16·4	+0·4
7209	110·632	6·692	+212·5	− 7·3	−225·9	− 1·3	+16·0	...
7213	110·736	6·796	+220·9	− 0·3	−230·8	− 4·7	+18·8	+2·8
7217	111·675	7·735	+231·0	+14·3	−224·5	− 0·6	+14·6	−1·4
7232	112·619	8·679	+197·0	+14·1	−211·8	− 27·0	+15·9	−0·1
7234	112·785	8·845	+168·8	− 7·4	−212·5	− 37·2	+16·6	−0·6
7243	113·647	9·707	+110·9	− 6·1	...	...	+17·9	+1·9
7264	114·769	10·829	+ 21·9	− 0·5	...	...	+17·8	+1·8
7280	121·680	3·326	+ 11·3	− 1·8	...	...	+14·5	−1·5
7282	124·689	6·335	+208·4	− 4·1	−246·6	− 27·7	+13·5	−2·5
7284	126·667	8·313	+194·8	− 4·5	−209·2	− 4·2	+15·1	−0·9
7289	130·640	12·286	−102·1	+ 1·8	+164·6	+ 4·3	+17·4	+1·4
7294	130·780	12·426	−108·3	+ 6·0	+182·1	+ 11·0	+15·4	−0·6
7301	132·635	14·271	−207·1	−17·5	+194·6	− 67·3	+17·6	+1·6
7302	132·808	14·444	−197·6	− 8·0	+241·4	− 20·5	...	...
7304	133·633	0·865	−150·0	+20·3	+169·0	− 69·6	+16·5	+0·5
7330	136·697	3·929	+ 67·6	+ 0·8	...	...	+15·3	−0·7
7335	137·667	4·899	+146·7	+ 4·1	−236·8	−101·7	+18·4	+2·4
7385	145·661	12·893	−144·7	− 0·1	+179·7	− 28·3	+15·9	−0·1
7392	148·670	1·488	−144·9	− 7·3	+184·4	− 15·2	...	...
7403	149·664	2·482	− 68·9	− 6·8	+173·5	+ 64·2	+19·4	+3·4
7412	150·658	3·476	+ 25·6	− 1·1	...	...	+17·0	+1·0

In order to obtain the best possible values of the elements and to secure at the same time a measure of their relative accuracy, the preliminary elements obtained graphically from the velocity curve were corrected by least squares, Schlesinger's method* being used. Corrections were carried through for the eccentricity, the semi-amplitude of velocity, the longitude of the apse, and the velocity of the system. Owing to the short interval covered by the observations, a correction was also applied for the period. No correction for the time of periastron passage was attempted, as with such a small eccentricity it is impossible to obtain corrections for both ω and T. With a correction for the period included it is not possible to combine observations of different cycles into normal places, so each single observation formed a normal place except on the four nights when two observations were obtained, where normal places were formed from the weighted mean velocity and phase of the two observations.

The preliminary and final elements are as given in the following table :—

Elements of 6° 1309.

Element.	Preliminary.	Final.
Period, P	14·4	14·414 ±0·0157 day
Eccentricity, e	0·04	0·0349 ±0·0114
Semi-amplitude primary, K_1	205·0	206·38 ±1·95 km.
Longitude of apse, ω	180°	181°·95 ±2°·54
Time of periastron passage, T	3031·870	3031·870 J.D.
Velocity of system, γ	+22·0	+23·94 ±1·4 km.
Semi-amplitude secondary, K_2	250·0	246·7 km.

When the first six of these elements had been determined from the primary spectrum, they were considered, with the exception of the amplitude and longitude, as defined also for the secondary. The longitude of the secondary is, of course, 180° distant from the primary, and from the plotted velocity measures of the secondary K_2 was estimated as 250 km. As these measures were of very different weights, a correction was determined by least squares, giving K_2 a value of 246·7 km.

The Mass of the System.

From the elements by well-known formulæ the projected semi-axis major and the masses were computed, resulting as follows :—

$$(a_1 + a_2) \sin i = 89{,}750{,}000 \text{ km.} = 129 \text{ solar radii}$$
$$m_1 \sin^3 i = 75{\cdot}6 \odot$$
$$m_2 \sin^3 i = 63{\cdot}3 \odot$$
$$(m_1 + m_2) \sin^3 i = 138{\cdot}9 \odot$$

These numbers evidently represent minimum values of the masses, and must be divided by $\sin^3 i$. As the average value of this quantity is about 0·6, the actual masses may be considerably larger. Indeed,

* *Pub. A.O.*, **1**, 33.

considerations later discussed show that the inclination of the orbit plane cannot likely be greater than 73°, and consequently the actual masses are at least 14 per cent. greater, making the minimum mass of the system nearly 160 times the sun.

The maximum masses hitherto determined from binary systems are—

V Puppis *	Spectrum	B1, B3	$m_1 + m_2 = 33\ \odot$
Y Cygni †	,,	B2, B2	$m_1 = 16{\cdot}6\ \odot \quad m_2 = 15{\cdot}3\ \odot$
Boss 6142 ‡	,,	Bp	$m_1 \sin^3 i = 18{\cdot}5\ \odot$
			$m_2 \sin^3 i = 12{\cdot}7\ \odot$
29 Can. Maj. §	,,	Oe	$m_2^3 \sin^3 i/(m_1 + m_2)^2 = 4{\cdot}58\ \odot$

For V Puppis and Y Cygni the masses are actual values, as for these eclipsing variables i is known. Boss 6142 and 29 Can. Maj. contain the factor $\sin^3 i$ just as 6° 1309, and the actual masses are subject to the same uncertainty. In 29 Can. Maj. only one spectrum appears, and only the mass function $m_2^3 \sin^3 i/(m_1 + m_2)^2 = 4{\cdot}58\ \odot$ is known. If the masses are equal, then $m_1 \sin^3 i$ and $m_2 \sin^3 i$ each become 18·2 times the sun, but if m_1 is greater than m_2 its mass is correspondingly increased. This was evidently the most massive known system until 6° 1309, but under any probable ratio of m_1 and m_2 will not approach the latter in mass.

The mass of 6° 1309 is then about four times that of any previously known stellar system, and, with the very high temperature and surface luminosity of the Oe and Oe5 stars, must be exceedingly bright. It will be of interest to see if we cannot obtain some estimate of its absolute magnitude and parallax.

The Absolute Magnitude.

The most promising method of attack seems to consist in applying probable values of surface brightness and density to these known masses. For the relative surface brightness of stars of different types Russell‖ has compiled several methods of attack, reducing all to a constant factor K which, when multiplied by the differences of standard colour-index, gives the difference of surface brightness in stellar magnitudes. He adopts for K a mean value of 4·0, which would give a surface brightness to an Oe star of about −3·4 magnitudes. When, however, the apparent diameters of the three late-type stars measured by the interferometer are compared with the computed results, it is found that a value for K of 4·8 gives the best agreement.¶ This value makes the surface brightness of an Oe star −4·0 magnitudes as compared with the sun. Seares,** in a recent exhaustive and valuable paper, finds values at least a magnitude lower for the B-type stars, although agreeing well with the later types. His results are, however, based solely on the theoretical laws of radiation and on Wilsing's stellar temperatures. As it is generally agreed that these temperatures for the B stars are considerably too low, probably proportionately more so for O-type, and as a relatively moderate increase of

* *Harvard Annals*, **84**, 157.
† *Pub. D.A.O.*, **1**, 213.
‡ *R.A.S.C.*, **10**, 297.
§ *Ibid.*, **11**, 343.
‖ *Pub. A.S.P.*, **32**, 307.
¶ *Ibid.*, **34**, 92.
** *Ap. J.*, **55**, 196.

assumed temperature would bring the luminosities into agreement, I prefer to accept Russell's corrected results based on colour-indices, and to assume for the high-temperature O-type star 6° 1309, a surface luminosity four magnitudes brighter than the sun.

Estimates of the density of such a star are also subject to some uncertainty. We have fairly reliable data on the density of early B-type stars from eclipsing variables where the average density is about 0·05 times the sun, varying between 0·0004 and ·18. The star under consideration, however, is not only several times as massive, but is also at a considerably higher temperature. The ratio $(1-\beta)$ of the radiation pressure * to the total pressure is for a star of mass 76 ⊙ about 0·83, and this should tend to cause it to "blow out," become much less dense, so that its density is probably considerably lower than 0·05. Further evidence pointing to lower density may be obtained from the consideration of the probable density of β Orionis. The spectral type is B8, the parallax 0″·007 is reliable, and the absolute magnitude −5·5, of the same order as 6° 1309, is among the largest known. It seems certain that the mass of β Orionis is exceptionally large, and it may be taken as 50 times the sun. The apparent diameter computed from Russell's corrected values † is 0″·0019, giving a density 0·002 times the sun, which cannot be far from an upper limit for this star. If we assume, as is probable, that β Orionis is in the giant stage, the density of 6° 1309 will be somewhat greater. Considering Seares' values of density in table xiv. ‡ and extrapolating for the greater mass and earlier type, it would seem that 0·01 is the most probable value of the density of the exceptional star 6° 1309. This agrees closely enough with the values of eclipsing variables when the exceptional mass is taken into account, and should lead to a determination of absolute magnitude not far astray.

With a mass 75·6 times, a density 0·01 times and a surface brightness −4·0 compared with the sun, the absolute magnitude of the brighter component 6° 1309 is −5·65, slightly greater than for β Orionis, which, with a visual magnitude of 6·71, corresponds to a parallax of 0″·00035, about 3000 parsecs, 10,000 light years. It is of interest to compare this value with one deduced from Eddington's theoretical considerations.§

For $m_1 = 75{\cdot}6\ \odot$,	$1-\beta = 0{\cdot}827$,	$M_1 = -6{\cdot}25$
For $m_2 = 63{\cdot}3\ \odot$,	$1-\beta = 0{\cdot}812$,	$M_2 = -6{\cdot}03$
For the system		$M_1 + M_2 = -6{\cdot}9$

These are bolometric magnitudes, and the corresponding correction to visual magnitudes for a star of this type ‖ is about +0·6 magnitudes. This brings the absolute magnitudes computed by the two methods into exact agreement, making the absolute magnitude of the system −6·3; and although too much weight should not be given to this coincidence, it should serve to strengthen our confidence in the substantial accuracy of the values of the density, absolute magnitude, and parallax deduced. A further deduction from the value of the density adopted might be

* *Ap. J.*, **48**, 210.
† *Pub. A.S.P.*, **34**, 92.
‡ *Ap. J.*, **55**, 202.
§ *Ap. J.*, **48**, 211.
‖ *M.N.*, **77**, 605.

noted as tending to show that too high values of absolute magnitude have not been obtained. With a density of 0·01 the diameter of the bright star is nearly 20 and of the faint 18 times the sun, while their separation is 65 solar diameters. It is practically certain that the two stars do not eclipse one another, as a variation every seven days in a sixth-magnitude star could hardly escape detection. Hence the inclination of the orbit plane cannot be greater than 73°, making $\sin^3 i = 0{\cdot}875$, and the minimum value of the masses 14 per cent. greater than the given figures or $m_1 = 86{\cdot}4$, $m_2 = 72{\cdot}3$, and the total mass nearly 160 times the sun. These larger values of the mass should result in larger absolute magnitudes and smaller parallax, but it is preferable to adhere to the values already given and merely consider this deduction as evidence that too great brightness or too low density has not been assumed.

The Stationary Calcium Lines.

One other point of interest in connection with this extraordinary system should be noted, the behaviour of the calcium lines. That they are absolutely stationary as previously stated and do not, as in some other binary systems, partially share in the oscillations of the other lines, cannot be doubted when their residuals given in the table of velocities are examined. Considered as stationary, the mean velocity is $+16{\cdot}0 \pm 0{\cdot}25$ km. per second, making the probable error of the determination of the velocity of H and K from a single plate only $\pm 1{\cdot}35$ km. per second. Such very accurate measures with single-prism dispersion and only two lines on a plate makes their fixity of position certain. Young's * conclusion that the calcium vapour in the case of stationary H and K surrounds the binary system may be open to question in this case, for unless at a very great distance the calcium vapour would probably have sufficient rotational motion with such a massive high-velocity internal binary system to show quite marked oscillations. It is, to say the least, a curious coincidence that the component of a solar motion of 20 km. per second is, in this direction $+15{\cdot}9$ km., practically exactly the numerical velocity of the calcium vapour. In other words, the latter is stationary with respect to the system of stars from which the solar motion is obtained. The similar condition in the case of most of the novæ and of many other spectroscopic binaries serves to make this appear more than a coincidence, notwithstanding difficulties in the way of a hypothesis of stationary clouds in space.

The systematic velocity of the binary system is $+23{\cdot}9 \pm 1{\cdot}4$ km. per second, nearly 8 km. more positive than that given by calcium. This difference is in the direction of the relativity displacement of the spectrum lines, but is considerably too great in magnitude. With the determined value of mass and density we get the gravitation potential at the surface of the brighter component 75·6 divided by 19·5 or 3·9 times that of the sun, equivalent to a velocity of $3{\cdot}9 \times 0{\cdot}635$ or $+2{\cdot}5$ km. per second. To be added to this would be the effect of the other star at a separation of 130 solar radii adding about $+0{\cdot}3$ km., making

* *Pub. D.A.O.*, **1**, 219.

a total displacement of +2·8 km. as compared with 8 km. The calcium lines would, of course, be shifted in the same direction, but the probable great distance of either a surrounding or an isolated cloud would make a negligible reduction, certainly not more than one- or two-tenths of a kilometre. In order that the relativity shift should equal the difference in velocity for the given masses, a density of the brighter component of nearly 0·4 times the sun would be required, a quite inadmissible value.

It cannot be said, therefore, that the different velocities of the binary system and of the calcium vapour offer evidence of much weight in favour of the relativity shift of the spectrum lines. If the vapour surrounds the binary and has the same space velocity, the measured difference, though in the right direction, is too great to be accounted for in this way. If, on the other hand, there is a stationary calcium cloud somewhere in the line of sight, and if we grant that the space velocity of such a massive O-type star would be small, still, all we can say as before is that the difference in velocity is in the right direction.

Summary.

We may summarise, in conclusion, the principal known and probable dimensions of this interesting system. It consists of two very bright Oe stars, 10,000 light years away, revolving around one another in an elliptic orbit of eccentricity ·035 in a period of 14·414 days at a separation of some 90,000,000 km. The brighter of the two stars has a probable mass of at least 86 times the sun, a density of 0·01, and an absolute magnitude −5·65. The fainter has a mass 72, the same density, and an absolute magnitude −5·4, the absolute magnitude of the system being −6·3, the greatest known. The velocity of the system is +23·9 km. and that of the calcium vapour +16·0 km. This difference, though in the right direction, is too great to be explained by the relativity displacement of the spectrum lines.

Dominion Astrophysical Observatory,
Victoria, B.C.:
1922 *May.*

---

*On the Light Changes and Other Peculiarities of Nova Persei No. 2 (Anderson, 1901).** By E. E. Barnard. (Plate 12.)

The following observations are a continuation of those printed in *Monthly Notices* for June 1920 (80, 710). For some time the star has kept bright, but at the last observations it had suddenly declined to the 14th magnitude. Unfortunately cloudy weather prevented a close watch of this important change. The observations of March 21 and April 15 were good, so that the sudden decline in brightness is real.

During the present season the light of the nova has had many ups and downs, with a strong tendency to keep bright. For many months

* With the 40-inch refractor of the Yerkes Observatory.

it remained at a fairly constant brightness, at or near $12^{m}\cdot5$. Several times it has started downward only to recover itself and become brighter again. Its maximum in all cases seems to be between 12 and 12·5 magnitude, never becoming as bright as 12th magnitude on the adopted scale—that of Seares at Mount Wilson. The comparisons have always been made with a magnifying power of 460.

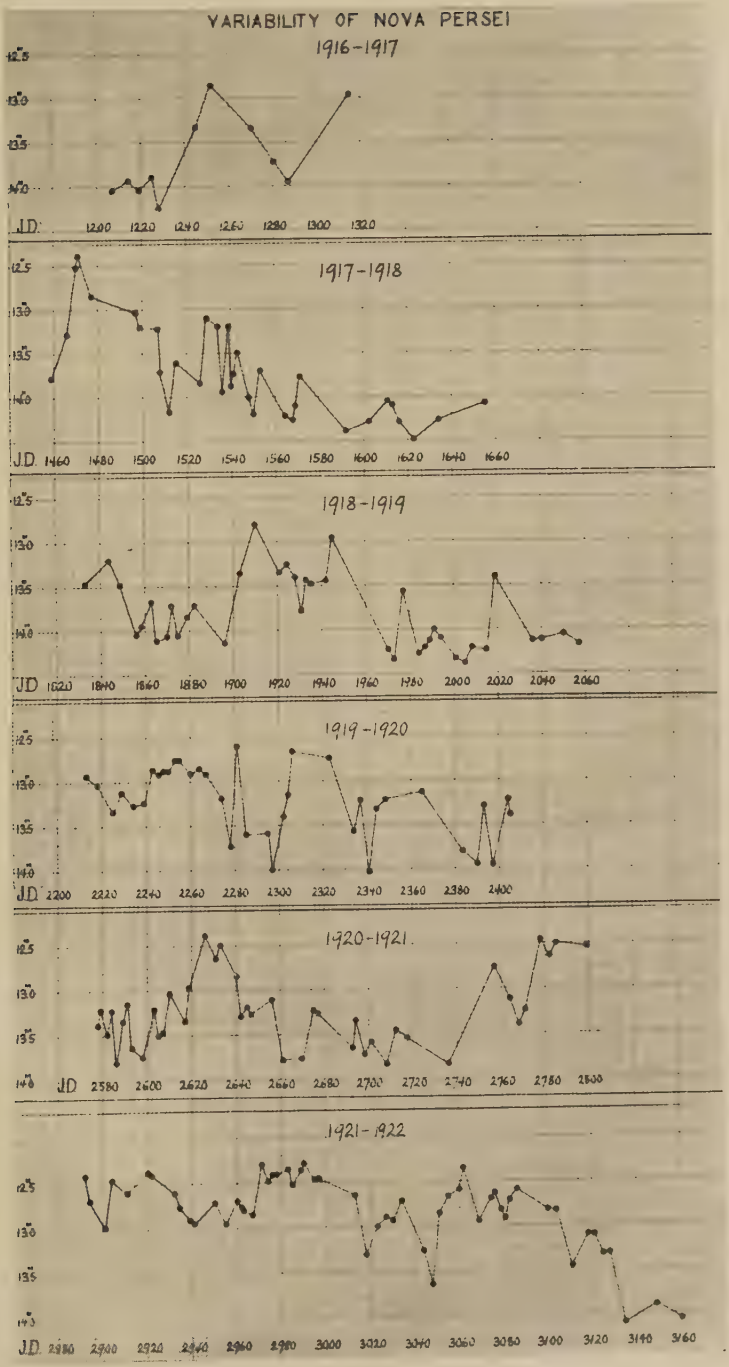
None of the novæ which have become faint, either at the end of their decline or near it, are so fascinating as Nova Persei. The others exhibit interesting features that in the main are much alike. Nova Persei, however, stands pre-eminent in its peculiarities, which can be set under two heads:—

1. Surrounding, moving nebulosity.
2. Variability in its light.

The first of these is well known from the remarkable photographs of Ritchey and Perrine, which showed the nebulous matter in rapid recession from the star following the outburst in 1901. A later nebulosity, that of 1916 December 16, has been shown by Lampland (whose photographs with the 40-inch reflector of the Lowell Observatory are so important in showing changes in the nebulæ) to be most interesting. His photographs of the phenomenon of the present nebulosity show changes in its form and a very slow recessional motion from the star.

But perhaps the most interesting phenomenon now connected with the nova is its remarkable variability. Professor Pickering had shown that previous to its great outburst it was a small variable star of about the 14th magnitude. Since its decline to its original brightness its light fluctuations have been extremely puzzling. Because of the importance of the subject, I have kept a very close watch upon the changes in brightness of the star in the past six years. These observations reveal a most remarkable condition of variability, in which there seems to be no definite order or period in the light changes. In adding the observations of two more seasons to my previous ones (*M.N.*, 80, 710), I have considered that it would bring out more clearly the peculiarities if the entire series was plotted, season by season. The result (Plate 12) is very striking. It shows the probable hopelessness of reconciling the changes with any definite period. Professor Turner has endeavoured to connect up the present variations of the nova with those before 1901. Unfortunately the pre-nova observations do not seem sufficient for this purpose. If he is able to make such a connection, it will be of the very greatest value in showing what effect the outburst had on the regular habits of the star. It is to be hoped the present observations may aid him in this important work.

In regard to the observations of 1919 November 15 (13·74), 18 (12·61), and 22 (13·61), (*M.N.*, 80, 714), it would almost seem as if one observation might be erroneous. This is not the case, however, for the change was real. It was specially noted on the 18th that the nova was brighter. The change attracted the eye at once, as also did the decline on the 22nd. Similar sudden changes are all real. There should be no hesitation in accepting them as such.


VARIABILITY OF NOVA PERSEI
1916-1917
12.5
13.0
13.5
14.0
J.D.
1200
1220
1240
1260
1280
1300
1320
1917-1918
12.5
13.0
13.5
14.0
J.D.
1460
1480
1500
1520
1540
1560
1580
1600
1620
1640
1660
1918-1919
12.5
13.0
13.5
14.0
JD
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000
2020
2040
2060
1919-1920
12.5
13.0
13.5
14.0
J.D.
2200
2220
2240
2260
2280
2300
2320
2340
2360
2380
2400
1920-1921
12.5
13.0
13.5
14.0
JD
2580
2600
2620
2640
2660
2680
2700
2720
2740
2760
2780
2800
1921-1922
12.5
13.0
13.5
14.0
J.D.
2880
2900
2920
2940
2960
2980
3000
3020
3040
3060
3080
3100
3120
3140
3160

It should be stated that the image of the nova in the large telescope (unlike that of Nova Aquilæ, Nova Cygni 1920, Denning, etc.), is stellar, and much like the other stars near it.

Another comparison star, No. 21, has been measured with reference to No. 15.

1920·845	Nov. 4	164°·86	151″·29
·864	11	164·82	151·18
·869	13	164·94	151·35
·878	16	164·99	151·17
·883	18	164·95	151·31
·908	27	165·08	151·08
1921·012	Jan. 4	165·01	151·50
1920·894		164·95	151·27

Professor Seares has supplied me with the magnitude of star 21 :—

Photo-visual mag. = 12·69 ; colour-index = +0·55.

The nova is not close enough to any good comparison star for direct measures of position. One of the comparison stars for the light of the nova, No. 14, was, in 1920–21, compared with B.D. + 43° 739 ($9^{m}\cdot0$).

B.D. + 43° 739 *and No.* 14.

1920·711	Sept. 16	352°·33	169″·16
·716	18	352·47	169·43
·724	21	352·21	169·05
·768	Oct. 7	351·67	169·24
1921·012	Jan. 4	352·34	169·22
·913	Nov. 29	352·45	169·05
1920·977		352·24	169·19

The bright star is Bonn A.G.C. 2948 ($9^{m}\cdot0$). These measures give for the position of star 14 :—

$$1920\cdot0\ \alpha\ 3^{h}\ 25^{m}\ 26^{s}\cdot64,\quad \delta + 43^{\circ}\ 39'\ 53''\cdot2.$$

The nova will again be compared with No. 14, but, taking the measures of 1902–3,

Nova and No. 14.

1902·557	300°·51	236″·47	7*n*

we get for it (the nova) the closely approximate position

$$1920\cdot0\ \alpha\ 3^{h}\ 25^{m}\ 45^{s}\cdot41,\quad \delta + 43^{\circ}\ 37'\ 53''\cdot2.$$

Magnitude Observations of Nova Persei.

Date.	C.S.T.*	J.D.	Mag.	No. of Comps.	Date.	C.S.T.*	J.D.	Mag.	No. of Comps.
1920	h m	242			1921	h m	242		
Sept. 10	13 30	2578	13·39	8	Feb. 15	6 30	2736	13·84	10
11	13 55	2579	13·22	9	Mar. 8	8 40	2757	12·75	9
14	13 50	2582	13·49	13	15	7 40	2764	13·12	9
16	13 45	2584	13·22	13	19	8 10	2768	13·41	7
18	13 0	2586	13·81	10	22	7 40	2771	13·24	9
21	13 15	2589	13·34	8	29	7 20	2778	12·46	9
23	13 15	2591	13·15	9	Apr. 2	7 35	2782	12·64	15
25	13 50	2593	13·63	7	5	7 30	2785	12·50	11
30	11 30	2598	13·74	9	19	8 30	2799	12·53	5
Oct. 5	13 10	2603	13·21	10	July 21	13 30	2892	12·41	7
7	12 40	2605	13·50	9	23	13 50	2894	12·68	8
9	11 50	2607	13·47	9	30	13 25	2901	12·97	10
12	11 20	2610	13·03	10	Aug. 2	13 50	2904	12·46	8
19	11 0	2617	13·35	10	9	13 55	2911	12·59	6
21	11 0	2619	12·97	9	18	13 15	2920	12·37	4
28	13 50	2626	12·38	5	20	13 0	2922	12·41	6
Nov. 2	11 55	2631	12·64	9	30	12 0	2932	< 12·6	
4	10 50	2633	12·49	8	Sept. 1	11 50	2934	12·76	7
11	11 35	2640	12·84	8	6	12 55	2939	12·91	7
13	11 0	2642	13·29	9	8	12 35	2941	12·94	7
16	8 40	2645	13·19	11	17	12 40	2950	12·72	9
18	10 40	2647	13·27	12	22	12 55	2955	12·95	9
27	10 20	2656	13·11	12	27	12 0	2960	12·70	7
Dec. 2	10 30	2661	13·78	19	29	14 0	2962	12·77	5
11	10 10	2670	13·66	6	30	11 15	2963	12·81	7
11	12 30	2670	13·88 †	12	Oct. 4	11 55	2967	12·86	9
16	11 20	2675	13·23	9	8	11 40	2971	12·30	6
18	11 0	2677	13·27	10	11	13 40	2974	12·49	4
1921					13	12 55	2976	12·41	7
Jan. 3	6 15	2693	13·65	11	15	9 50	2978	12·41	7
4	6 35	2694	13·35	13	20	11 30	2983	12·36	7
8	12 40	2698	13·73	13	22	10 20	2985	12·53	7
11	6 20	2701	13·58	12	26	9 50	2989	12·37	3
18	6 15	2708	13·84	7	27	13 15	2990	12·29	9
22	11 0	2712	13·45	11	Nov. 1	11 15	2995	12·47	7
27	12 15	2717	13·54	12	3	13 30	2997	12·46	5

* C.S.T., $6^h\ 0^m$ slow of G.M.T.
† Mean = Dec. 11, $11^h\ 20^m$, $13^m{\cdot}77$.

Magnitude Observations of Nova Persei—continued.

Date.	C.S.T.*	J.D.	Mag.	No. of Comps.	Date.	C.S.T.*	J.D.	Mag.	No. of Comps.
1921	h m	242			1922	h m	242		
Nov. 19	11 00	3013	12·65	10	Jan. 21	7 25	3076	12·63	13
24	11 40	3018	13·31	11	24	7 55	3079	12·82	8
29	8 25	3023	13·00	10	26	10 45	3081	12·92	12
Dec. 3	11 35	3027	12·89	9	28	7 5	3083	12·71	8
6	7 55	3030	12·93	10	31	8 45	3086	12·59	12
10	7 15	3034	12·71	9	Feb. 14	7 15	3100	12·82	18
20	8 50	3044	13·27	10	18	7 45	3104	12·83	11
24	8 45	3048	13·64	12	25	8 45	3111	13·45	11
27	8 10	3051	12·85	10	Mar. 4	8 25	3118	13·10	10
31	11 5	3055	12·67	10	7	8 20	3121	13·11	11
1922					11	8 40	3125	13·32	9
Jan. 5	10 30	3060	12·59	9	14	8 50	3128	13·31	9
7	6 35	3062	12·35	2	21	8 10	3135	14·09	14
14	9 45	3069	12·94	11	Apr. 4	8 25	3149	13·9	6
19	10 30	3074	12·70	15	15	7 40	3160	14·06	14

Notes.

1920 Dec. 11. Uncertain.
1921 Aug. 30. Glimpsed through clouds.
Oct. 11. Uncertain; clouds.
1922 Jan. 7. Uncertain; clouds.
Apr. 4. Uncertain in haze.

Yerkes Observatory,
Williams Bay, Wisconsin:
1922 *May* 6.

Note on the Parallaxes of Open and Moving Clusters.

By Peter Doig.

The chief methods for estimation of distances of open clusters are based upon:—

(*a*) The discussion of proper motions of the constituent stars, such as those whereby apparent converging or diverging motions, which are assumed really parallel and equal, or apparent contraction or expansion of the angular area occupied, can be made to give parallaxes, by the aid of spectroscopic radial velocities.

(*b*) The study of the luminosity curves (or relative numbers of stars of successive grades of luminosity), an assumption being made that these are the same for the stars in the clusters as that so far adopted for space generally.

(*c*) The estimate of probable absolute magnitude based on empirical relationships between luminosities and spectral classes, Giant and Dwarf classification being taken into account.

Some results of methods (*a*) and (*b*) are shown below :—

TABLE I.

Parallaxes of Clusters.

Cluster.	Method (*a*).*	Method (*b*).
	"	"
Perseus moving cluster	0·010	...
Præsepe	0·008	0·024 †
Pleiades	0·014	0·036 ‡
Taurus Group	0·025	...
Orion Group	0·007	0·0054 §

It will be observed that the parallaxes by method (*a*) for Præsepe and the Pleiades (which are means of results that are tolerably consistent *inter se*) are considerably smaller than those due to method (*b*).

The third method (*c*) has not been systematically employed to any considerable extent, but seems to the writer to be worthy of weight at least equal to that given to method (*b*).

On the reasonable assumption that the most frequent absolute magnitude for each spectral class will be about the same in the cluster as that which is found in space generally for the Giant and Dwarf of the same spectrum, parallaxes are derivable from the *differences* between visual magnitudes of the stars in the cluster and the most probable absolute magnitudes. A criterion of the relative values of the results tabulated above is thus provided, and also a method by which it is possible to make a rapid estimation of the mean parallax of groups of stars of common proper motion for which spectra are available. In the case of very distant and faint telescopic clusters of the general form and density of the groups here considered, of which many are known, or even in the case of the globular clusters, when photography has extended to include the fainter stars therein contained, estimates of distance may be possible by the use of "colour indices" and derived spectral classification, due allowance being made for the differences which appear to exist in these indices for Giants and Dwarfs of the same spectral type. It is possible, however, that the direct use of the most frequent absolute magnitudes for Giants and Dwarfs of particular colour indices, as derived from stars in the solar neighbourhood, will be preferable in these cases.

Table II. shows the most frequent magnitudes for Giants and Dwarfs

* The values in this column are means of those given in the collected results presented in Rasmuson's paper, "A Research on Moving Clusters," *Meddelanden från Astronomiska Observatorium*, No. 26 (1921).

† Schouten, *Observatory*, **42**, 240 (1919).

‡ Schouten, *ibid.*, **42**, 112.

§ Kapteyn, *Mount Wilson Contributions*, **147**, 62 (1918), or ***Ap. Jnl.***, **47** (1918).

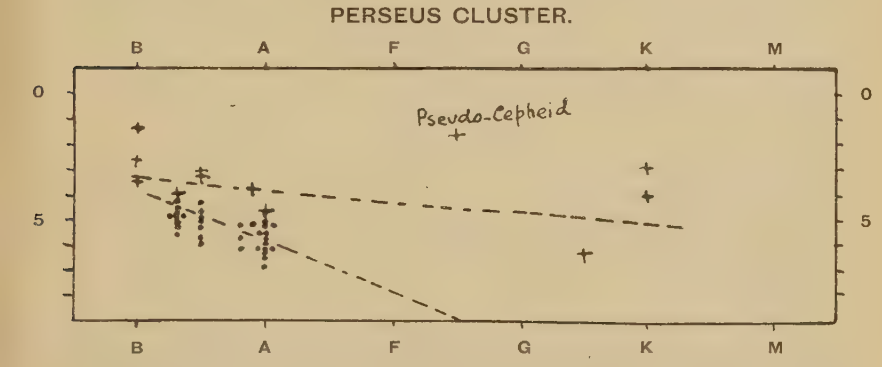
of space generally. It is based on Adams's list* of 1646 stars and on 103 from other sources.

TABLE II.

Absolute Magnitudes (M).

Spectra.	Giants.	Dwarfs.
Bo	−1·7	−1·2
Ao	−1·2	+0·7
Fo	−0·8	+2·5
Go	−0·4	+4·4
Ko	+0·1	+6·3
M	+0·5	+9·5?

In the figure the spectra and visual magnitudes (reduced to mean proper motions) for the moving cluster in Perseus have been shown



graphically. The broken lines are the above maximum frequency values fitted to what appears to be the most suitable configuration as Giants and Dwarfs of the stars in the group, crosses indicating those which are probably Giants. In this case $(M - m)$, where m is visual magnitude, has the value $-5{\cdot}1$, whence from the expression $\log \pi = \frac{(M-m)-5}{5}$ we obtain a parallax of $0''{\cdot}0096$. This is in good agreement with the values found by Rasmuson ($0''{\cdot}011$) and Bottlinger ($0''{\cdot}009$). The mean of the trigonometrical and spectroscopical parallaxes of seven of the stars is, however, $0''{\cdot}014$.† The data of the figure may be presented as in Table III. A similar graphical procedure has been gone through for several clusters,‡ and the results are shown in Tables IV. to VII.

* Adams and others, *Mount Wilson Contributions*, No. 199, or *Astrophysical Journal*, 53 (1921). See also Seares, *Ap. Jnl.*, 54 (1922 April); *B.A.A. Jnl.*, 32, 2, 59, and *M.N.R.A.S.*, 82, 6, 374.

† Rasmuson, *loc. cit.*, p. 30.

‡ Data from Rasmuson, magnitudes reduced to mean proper motion, except in the case of the Pleiades, for which *Lick Observatory Bulletin*, No. 333 (1921), has been used.

In the tables, m_m is mean visual magnitude of the stars for each division as graded, M_m is the mean absolute magnitude corresponding to the parallax obtained, and M is the most frequent absolute magnitude of a star of the mean spectral type of each division of the table. The number of the stars in each division and the mean spectral type are also shown.

Table IV. Cluster in Coma Berenices.—For this cluster W. H. Pickering * obtained 0″·0109 from a study of luminosities.

Table V. The Taurus Group.—The parallax obtained, 0″·019, is less than that given by method (*a*), 0″·025, but does not differ greatly from the mean of the spectroscopic and trigonometrical parallaxes for 13 stars, 0″·021.†

Table VI. The Pleiades.—Trumpler,‡ who has made a special study of this cluster, considers a value of about 0″·010 to be about the best figure for the parallax. The mean of the hypothetical parallaxes of nine connected pairs of stars in the cluster is 0″·010§; five trigonometrical parallaxes give 0″·014; Pickering‖ (0″·005), Hertzsprung ¶ (0″·0063), Russell ** (0″·0063) and Trumpler ‡ (0″·008), get somewhat smaller values from luminosities. Schouten's value, 0″·036, derived from the "luminosity law," seems at least three or four times too great.

Table VII. Præsepe and Associated Stars.—The parallax obtained, 0″·0083, agrees well with those found by method (*a*), Schwarzschild, 0″·0063, Rasmuson, 0″·011, Kohlschutter 0″·0072, but is less than the mean spectroscopic and trigonometrical value for three stars, 0″·014.†† The value by method (*b*) due to Schouten, 0″·024, which would mean abnormally small luminosities for the A type stars, is therefore not supported.

It may be of interest to note the bearing of this short investigation on the controversy regarding the distances of globular clusters. Shapley and others favour values from six to ten times as great as those advanced by Schouten, Curtis, and other investigators, which are largely based on luminosity law determinations (method (*b*) above). As parallaxes by this method appear to be something like three or four times too large for the nearer clusters, distances of the globular ones similarly derived are also likely to be too small. The discrepancy may be partly due to a real want of similarity between the luminosity curve of a cluster and that for space generally; it is also possible that the true luminosity curve for stars in the solar neighbourhood is not yet accurately known, effects of observational preference of the intrinsically brighter and remoter stars vitiating that utilised at the present time.‡‡

* *H.C.O. Circular*, 206 (1918).
† Rasmuson, *loc. cit.*, p. 14.
‡ *Publications Astronomical Society of the Pacific*, 32, 215 (1921).
§ *Observatory*, 45, 54 (1922), and *L.O. Bulletin*, No. 333.
‖ *H.C.O. Circular*, 206 (1918).
¶ *Ast. Nach.*, 195 (1913).
** *Proc. American Philosophical Society*, 51 (1912).
†† Rasmuson, *loc. cit.*, p. 23.
‡‡ See Crommelin's article, "The Distances of Globular Clusters," *Observatory*, 45, 139 (1922).

TABLE III.

Perseus Moving Cluster.

		"Earlier" than B6.	B6 to A5.	A6 to F5.	F6 to G5.	G6 to K5.
Giants.	m_m	3·0	4·3	1·7	6·4	3·5
	M_m	−2·1 } 6, B2	−0·8 } 2, A0	−3·4 } 1, F5	+1·3 } 1, G5	−1·6 } 2, K0
	M	−1·6 (*g*B2)	−1·2 (*g*A0)	−0·6 (*g*F5)	−0·1 (*g*G5)	+0·1 (*g*K0)
	$M_m - M$	−0·5	+0·4	−2·8	+1·4	−1·7
Dwarfs.	m_m	5·0	5·8			
	M_m	−0·1 } 16, B4	+0·7 } 17, A0			
	M	−0·5 (*d*B4)	+0·7 (*d*A0)			
	$M_m - M$	+0·4	0			

$(M - m) = -5·1$.
Parallax = 0″·0096.

Note.—The F5 type Giant is a Pseudo-Cepheid.

TABLE IV.

Coma Berenices Cluster.

		B6 to A5.	A6 to F5.	F6 to G5.	G6 to K5.	"Later" than K5.
Giants.	m_m				6·1	7·5
	M_m				+1·5 } 5, K0	+2·9 } 1, M
	M				+0·1 (*g*K0)	+0·5 (*g*M)
	$M_m - M$				+1·4	+2·4
Dwarfs.	m_m	6·4	6·9	8·8	8·5	
	M_m	+1·8 } 17, A1	+2·3 } 7, F2	+4·2 } 14, G1	+3·9 } 3, K1	$(M - m) = -4·6$.
	M	+0·9 (*d*A1)	+3·0 (*d*F2)	+4·6 (*d*G1)	+6·5 (*d*K1)	Parallax
	$M_m - M$	+0·9	−0·7	−0·4	−2·6	= 0″·012.

TABLE V.

The Taurus Group.

		B6 to A5.	A6 to F5.	F6 to G5.	G6 to K5.
Giants.	m_m	3·5		4·0	3·9
	M_m	−0·1 } 1, A5		+0·4 } 1, G0	+0·3 } 3, K0
	M	−1·0 (*g*A5)		−0·4 (*g*G0)	+0·1 (*g*K0)
	$M_m - M$	+0·9		+0·8	+0·2
Dwarfs.	m_m	5·0	6·0	5·7	
	M_m	+1·4 } 28, A2	+2·4 } 8, F0	+2·1 } 1, G0	$(M - m) = -3·6$.
	M	+1·1 (*d*A2)	+2·5 (*d*F0)	+4·4 (*d*G0)	Parallax
	$M_m - M$	+0·3	−0·1	−2·3	= 0″·019.

TABLE VI.

The Pleiades.

		"Earlier" than B6.	B6 to A5.	A6 to F5.	
Giants.	m_m	4·1 } 7, B6			$(M-m)=-6\cdot1$. Parallax $=0''\cdot006$.
	M_m	−2·0 }			
	M	−1·4 (gB6)			
	M_m-M	−0·6			
					F6 to G5.
Dwarfs.	m_m	5·5 } 1, B5	7·5 } 41, A0	9·2 } 3, F5	9·6 } 14, G0
	M_m	−0·6 }	+1·4 }	+3·1 }	+3·5 }
	M	−0·3 (dB5)	+0·7 (dA0)	+3·5 (dF5)	+4·4 (dG0)
	M_m-M	−0·3	+0·7	−0·4	−0·9

TABLE VII.

Præsepe.

		B6 to A5.	A6 to F5.	F6 to G5.	G6 to K5.
Giants.	m_m			5·1 } 2, G0	5·5 } 1, K0
	M_m			−0·3 }	+0·1 }
	M			−0·4 (gG0)	+0·1 (gK0)
	M_m-M			+0·1	0
Dwarfs.	m_m	6·6 } 6, A2	6·7 } 1, F0		
	M_m	+1·2 }	+1·3 }	$(M-m)=-5\cdot4$. Parallax $=0''\cdot0083$.	
	M	+1·1 (dA2)	+2·5 (dF0)		
	M_m-M	+0·1	−1·2		

On the Curvature of the Lines in the Prismatic Spectrum.
By H. C. Plummer, F.R.S.

1. The curvature of the image of a straight slit, produced by the passage of the light through a prism, or train of prisms, is a well-known effect. It has considerable importance in radial velocity work, where the relative displacement of the stellar and adjacent comparison spectra is most conveniently neutralised by using a suitably curved slit. In the spectro-heliograph the effect is marked, and cannot be avoided in quite the same way, since, if either slit be straight while the other is curved, a distortion of the solar image will clearly result. Here the curvature must be shared in equal proportion by both the slits. A discussion of this property of the prism was given by L. Ditscheiner * in 1865, and his paper has been commonly referred to for the main result in the text-books.† The proof is thus not always easily accessible,

* *Ber. d. Wiener Akad.*, **51** (2), 368–383.

† See, however, Kayser's *Handbuch der Spectroscopie*, **1**, 319, for further references.

and it may not be superfluous to consider the matter again on independent lines. Ditscheiner's notation will be retained as far as possible, in order to facilitate comparison with the original memoir.

2. Both the collimator lens and the camera (or telescope) lens are to be regarded as optically perfect. The effect of the prism can therefore

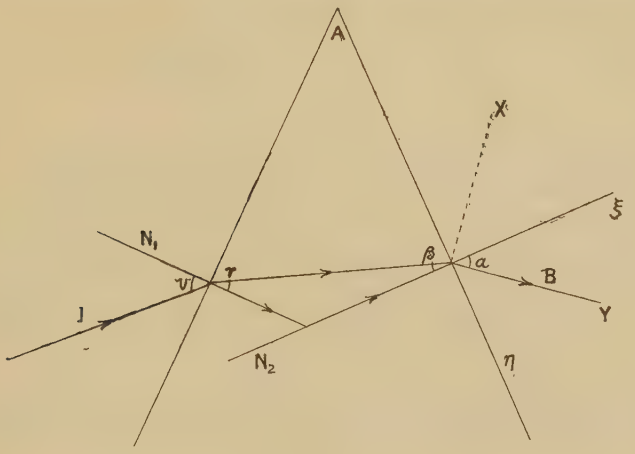


FIG. 1.

be found by considering those rays only which pass through the centres of these lenses, and a spherical representation of their directions is sufficient. Let N_1, N_2 represent the normals to the two faces of the prism, so that (fig. 2) $N_1N_2 = A$, the angle of the prism. The incident rays from the straight slit lie in a plane represented by the great circle

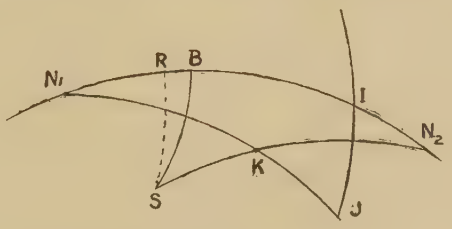


FIG. 2.

IJ at right angles to N_1N_2. The first refraction deviates the ray J to K, and the second refraction deviates the ray K to S. In the principal plane the corresponding joint effect is to deviate I to B.

In the triangle N_1IJ, $N_1IJ = 90°$. Let $N_1I = v$, $N_1J = n'$, $IJ = \theta$, $IN_1J = w'$. Then

$$\sin n' \sin w' = \sin \theta$$

$$\sin n' \cos w' = \sin v \cos \theta.$$

In the triangle KN_1N_2, $N_1N_2 = A$, $KN_1N_2 = w'$. Let $KN_1 = n$, $KN_2 = m'$, $KN_2N_1 = w$. Then

$$\sin m' \sin w = \sin n \sin w'$$
$$\sin m' \cos w = \sin A \cos n - \cos A \sin n \cos w'.$$

But, if μ is the refractive index of the prism,

$$\sin n' = \mu \sin n, \quad \sin m = \mu \sin m'$$

where $m = SN_2$. Hence

$$\sin m \sin w = \sin n' \sin w' = \sin \theta$$
$$\sin m \cos w = \sin A(\mu^2 - \sin^2 n')^{\frac{1}{2}} - \cos A \sin n' \cos w'$$
$$= \sin A(\mu^2 - \sin^2 \theta - \sin^2 v \cos^2 \theta)^{\frac{1}{2}} - \cos A \sin v \cos \theta.$$

These equations define the position of S in terms of v and θ, and contain the solution of the problem.

3. Let SR be the arc of a great circle at right angles to N_1N_2, so that

$$\sin SR = \sin m \sin w = \sin \theta$$

or $SR = \theta$. The inclination of the ray to the principal plane is unaltered by its passage through the prism, a fact well known. Since

$$\sin m \cos w = \sin N_2R \cos SR,$$

it follows that

$$\sin N_2R = \sin A\{(\mu^2 - 1)\tan^2 \theta + \mu^2 - \sin^2 v\}^{\frac{1}{2}} - \cos A \sin v;$$

while, on putting $\theta = 0$,

$$\sin N_2B = \sin A(\mu^2 - \sin^2 v)^{\frac{1}{2}} - \cos A \sin v.$$

Let $N_2B = \alpha$, this being the angle of emergence in the principal plane from the second face of the prism, and $N_2R = \alpha'$. Thus

$$\sin \alpha' - \sin \alpha = \sin A[\{\mu^2 - \sin^2 v + (\mu^2 - 1)\tan^2 \theta\}^{\frac{1}{2}} - (\mu^2 - \sin^2 v)^{\frac{1}{2}}],$$

which is an exact equation. At this stage an approximation is made. Thus if the slit be so short that $\tan^4 \theta$ may be neglected,

$$(\alpha' - \alpha)\cos \alpha = \tfrac{1}{2}(\mu^2 - 1)\sin A \tan^2 \theta\ (\mu^2 - \sin^2 v)^{-\frac{1}{2}},$$

and if $\sin v = \mu \sin r$, so that r is the angle of emergence at the first face of the prism,

$$BR = \frac{\mu^2 - 1}{2\mu} \cdot \frac{\sin A}{\cos \alpha \cos r} \cdot \theta^2.$$

Let f be the focal length of the camera lens. At the centre of the field the curvature of the image of the slit may be obtained by projecting on a concentric sphere of radius f and treating the small area

concerned as the image plane. Hence, if ρ is the radius of curvature of a line of the spectrum at the centre of the field,

$$2\rho \,.\, f \,.\, \mathrm{BR} = f^2 \,.\, \mathrm{SR}^2,$$

and therefore

$$\rho = \frac{\mu f}{\mu^2 - 1} \,.\, \frac{\cos\alpha \cos r}{\sin \mathrm{A}}.$$

In the case of minimum deviation, $\mathrm{A} = 2r$, $\alpha = v$, $\sin v = \mu \sin \frac{1}{2}\mathrm{A}$, and

$$\rho = \tfrac{1}{2} f \cot v \,.\, \frac{\mu^2}{\mu^2 - 1},$$

a result first found by Bravais.

Thus the curvature of the central line of the spectrum is obtained. If this is to be neutralised by giving the necessary compensating curvature to the slit, the corresponding result will be obtained by substituting for f the focal length of the collimator and for α, r the angles v, β which are the angles of incidence at the first and second faces of the prism. This follows by the principle of reversibility.

4. Rays from the straight slit passing centrally through the collimating lens lie in a plane. Those which pass centrally through the camera lens lie on a cone, of which the equation is found easily. Let the axis of z be normal to the principal plane of the prism, with an axis of ξ parallel to the normal N_2, and an axis of η perpendicular to the axis of ξ in the principal plane. Then

$$\xi/\mathrm{R} = \cos m, \quad \eta/\mathrm{R} = \sin m \cos w, \quad z/\mathrm{R} = \sin m \sin w = \sin\theta,$$

where (ξ, η, z) is any point on the refracted ray S and $\xi^2 + \eta^2 + z^2 = \mathrm{R}^2$. The refracted ray B in the principal plane, which is the axis of the camera lens or telescope and makes the angle α with the normal N_2, may be taken as the axis of y, with a perpendicular axis of x also in the principal plane. Then

$$\xi = x \sin\alpha + y \cos\alpha, \quad \eta = y \sin\alpha - x \cos\alpha, \quad \xi^2 + \eta^2 = x^2 + y^2.$$

The equation

$$\sin m \cos w = \sin \mathrm{A}(\mu^2 - \sin^2\theta - \sin^2 v \cos^2\theta)^{\frac{1}{2}} - \cos \mathrm{A} \sin v \cos\theta,$$

already obtained, can now be written

$$\begin{aligned}\eta &= \sin \mathrm{A}\{(\mu^2 - \sin^2 v)\mathrm{R}^2 - z^2\cos^2 v\}^{\frac{1}{2}} - \cos \mathrm{A} \sin v(\mathrm{R}^2 - z^2)^{\frac{1}{2}} \\ &= \sin \mathrm{A}\{(\mu^2 - \sin^2 v)(\xi^2 + \eta^2) + (\mu^2 - 1)z^2\}^{\frac{1}{2}} - \cos \mathrm{A} \sin v(\xi^2 + \eta^2)^{\frac{1}{2}}.\end{aligned}$$

In a rational form this gives the quartic cone

$$\begin{aligned}\{(\mu^2 \sin^2 \mathrm{A} - \sin^2 v)\xi^2 + (\mu^2 \sin^2 \mathrm{A} - \sin^2 v - 1)\eta^2 + (\mu^2 - 1)\sin^2 \mathrm{A} \,.\, z^2\}^2 \\ = 4 \cos^2 \mathrm{A} \sin^2 v \,.\, \eta^2(\xi^2 + \eta^2).\end{aligned}$$

The vertex of the cone is at the origin, which may be identified with the centre of the camera lens, and the cone passes through the intersection of

$$\xi^2 + \eta^2 = a^2,$$

which is a right circular cylinder with axis parallel to the edge of the prism, and

$$(\eta + a \cos A \sin v)^2 - (\mu^2 - 1) \sin^2 A \,.\, z^2 = a^2(\mu^2 - \sin^2 v) \sin^2 A,$$

which is a right hyperbolic cylinder with generators normal to the second face of the prism.

5. The image plane is $y = f$, and the equation of the central line of the spectrum in this plane can be deduced from the quartic cone by the substitution

$$\eta = f \sin \alpha - x \cos \alpha, \quad \xi^2 + \eta^2 = f^2 + x^2.$$

It is therefore

$$\{(\mu^2 \sin^2 A - \sin^2 v)(f^2 + x^2) - (f \sin \alpha - x \cos \alpha)^2 + (\mu^2 - 1) \sin^2 A \,.\, z^2\}^2$$
$$= 4 \cos^2 A \sin^2 v (f^2 + x^2)(f \sin \alpha - x \cos \alpha)^2.$$

This curve passes through the point $x = 0$, $z = 0$ necessarily; it is, in fact, easily verified that

$$\mu^2 \sin^2 A - \sin^2 v - \sin^2 \alpha = 2 \cos A \sin v \sin \alpha.$$

Near this point x^2 can be neglected in comparison with x, and thus the equation reduces to

$$(\mu^2 \sin^2 A - \sin^2 v - \sin^2 \alpha) f^2 + 2fx \sin \alpha \cos \alpha + (\mu^2 - 1) \sin^2 A \,.\, z^2$$
$$= 2f \cos A \sin v (f \sin \alpha - x \cos \alpha)$$

or

$$(\mu^2 - 1) \sin^2 A \,.\, z^2 + 2fx \cos \alpha (\sin \alpha + \cos A \sin v) = 0$$

or

$$(\mu^2 - 1) \sin A \,.\, z^2 + 2\mu fx \cos \alpha \cos r = 0.$$

This parabolic form naturally reproduces the expression for the radius of curvature found above.

6. The direction of the curvature may be considered at this point. In fig. 2 the points represent the directions *from* which the rays proceed, the directions of the normals to the two faces of the prism being as shown in fig. 1. Thus the form of the curve BS, which is bent from N_2 towards N_1, indicates that a straight line parallel to the edge and viewed simply through the prism will appear as though concave towards the edge of the prism. In order to appear straight, the line, represented by IJ, must itself be bent in the opposite direction. After the rays from a straight slit have passed through the camera lens the curvature is reversed, and the spectrum lines will be convex towards the edge of the prism, the curve running to the negative part of the axis of x as drawn in fig. 1, and as indicated by the last equation above.

Similarly the positive direction of the axis of η is parallel to the second face of the prism and downward from the edge, as shown in fig. 1. With the arbitrary constant a equal to f, the circular cylinder of § 4 may be identified with the image plane $y = f$. The trace of the hyperbolic cylinder on the (η, z) plane has its centre displaced upwards (or towards the prism edge) from the camera lens, and it is the lower branch of the hyperbola which gives the image of the slit.

7. The position of the telescope in which the lines of the spectrum possess minimum curvature involves a discussion which is perhaps more complicated than the importance of the question justifies. For grazing emergence from the prism on the one side $\alpha = 90^\circ$, $\rho = 0$, and the curvature is infinite. From this position $\cos\alpha \cos r$ increases and the curvature decreases. If the rays can be traced so far, when α and β become negative on the other side of the normal to the second face of the prism, $-\alpha$, $-\beta$ and $r = A - \beta$ increase towards grazing emergence, $\cos\alpha \cos r$ decreases, and the curvature increases. Hence minimum curvature, if it occurs at all, must occur before the normal is passed, with α and β positive. Further, $\alpha < 90^\circ$, $\beta < M$, and $|r| < M$, where M is the critical angle given by $\mu \sin M = 1$. Let $X = \cos\alpha \cos r = (\mu^2 - 1)\rho \sin A/\mu f$. Then

$$dX = -\sin\alpha \cos r \,.\, d\alpha - \cos\alpha \sin r \,.\, dr.$$

$$\cos\alpha \,.\, d\alpha = \mu \cos\beta \,.\, d\beta = -\mu \cos\beta \,.\, dr.$$

Hence

$$\mu \cos\beta \,.\, \frac{dX}{d\alpha} = \cos^2\alpha \sin r - \mu \sin\alpha \cos\beta \cos r$$

$$= \sin r - \mu^2 \sin\beta \cos(\beta - r)$$

$$= \sin(A - \beta) - \mu^2 \sin\beta \cos(2\beta - A)$$

$$= -(\mu^2 - 1)\cos A, \qquad \text{when } \beta = -90^\circ$$

$$= \sin A, \qquad \text{when } \beta = 0$$

$$= -\tfrac{1}{2}\mu^2 \sin 2M \cos(A - M), \quad \text{when } \beta = M$$

$$= \cos A(\mu^2 - 1), \qquad \text{when } \beta = 90^\circ.$$

If $A < 90^\circ$, these signs are alternate. There are three values of $\tan\beta$ which make the expression vanish, and it is only the smaller positive value of β, intermediate between 0 and M, which is both optically possible and capable of giving rise to minimum curvature of the lines of the spectrum.

8. This value of β, which of itself can be found to satisfy the conditions, requires a value of r intermediate between A and $A - M$. If $A < M$, this value necessarily lies between the values $\pm M$, and a position of minimum curvature certainly exists. If $A > M$, the greatest value of A giving rise to possible conditions is limited by $r < M$, where r is given by

$$\sin r - \mu^2 \sin(A - r)\cos(A - 2r) = 0.$$

This greatest value must therefore satisfy

$$\sin^3 M - \sin(A - M)\cos(A - 2M) = 0$$

or

$$\sin(3M - 2A) = \sin M \cos 2M.$$

A few corresponding values may be given:

$\mu = 1$	$\sqrt{1{\cdot}5}$	4/3	1·5	1·6	2·0
$M = 90^\circ$	$54^\circ{\cdot}8$	$48^\circ{\cdot}6$	$41^\circ{\cdot}8$	$38^\circ{\cdot}7$	$30^\circ{\cdot}0$
$A = 180$	90	75·6	60·6	54·2	37·8

If the angle of the prism exceeds this limit, the curvature of the lines is least when the position of normal emergence is approached as closely as possible. Ditscheiner, who worked several cases numerically for $\mu = 1{\cdot}5$, found the limit closely approached at $A = 60°$. When the angle of the prism is below the prescribed limit, and a true minimum curvature exists, the position is perhaps more easily found by trial from the equation

$$\tan(A - 2\beta) = (\mu^2 - 1)\tan\beta$$

than from the corresponding cubic in tan β, employed by Ditscheiner,

$$(\mu^2 - 1)\tan^3\beta - (2\mu^2 - 1)\tan A \tan^2\beta - (\mu^2 + 1)\tan\beta + \tan A = 0.$$

When $A > 90°$ this equation contains only one change of sign instead of two. There is therefore only one positive root, which still lies between 0 and M, since $A - M < 90°$. Hence the conditions of the problem are not materially altered by a very low index of refraction which permits an obtuse-angled prism to be employed. The curvature of the lines has at most one turning value, which occurs in all cases when A does not exceed a certain limit. It is easy to see that the corresponding position never coincides with the position of minimum deviation, and its attainment can be ignored as a factor in practical design. The curvature can only be mitigated by using lenses of long focal length, as Ditscheiner concluded.

The circumstances which lead to minimum curvature of the spectrum line are connected with another property. If D is the deviation corresponding to the angle of incidence v, it is easily shown that the dispersion is

$$\frac{dD}{d\mu} = \frac{\sin A}{\cos a \cos r} = \frac{\mu}{\mu^2 - 1} \cdot \frac{f}{\rho}.$$

The position of minimum curvature is therefore also the position of minimum dispersion.

On a Method of determining Photographic Star Magnitudes without the Use of Screens or Gratings. By J. Halm.

The method described in the following pages has been evolved from an investigation of the photometric properties of the photographic plate based on Abney's and Kron's experimental researches, of which an account has already been rendered in several papers published in this Journal. Its advantage lies in the fact that no screen or grating is required for fixing the scale, so that any photographic telescope or camera can be employed without further appliances than those used in ordinary star photography. The internal evidence of the plate furnishes all the data necessary for the determination of photometrically correct magnitudes throughout the whole range of stars within the optical power of the telescope, provided that the magnitude of one star is known from other sources.

I have shown in previous papers that the relationship between the intensity, I, incident on the plate, the time of exposure, t, and the size (or opacity) of the star's image, D, is expressible by an equation of the form

$$f(\mathrm{D}) = \mathrm{I}t/\psi \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $f(\mathrm{D})$ is an unknown function of D and

$$\psi = \tfrac{1}{2}(i^{\alpha} + i^{-\alpha})$$
$$i = \mathrm{I}/\mathrm{I}_0$$
I_0 = optimal intensity of the plate.

For intensities much smaller than I_0 we may write

$$f(\mathrm{D}) \propto \mathrm{I}^{(1+\alpha)}t \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or, expressed in logarithmic form,

$$\log f(\mathrm{D}) = \log t - \frac{1+\alpha}{2{\cdot}5}m + \text{constant} \quad . \quad . \quad . \quad (3)$$

since

$$\log \mathrm{I} = -0{\cdot}4m + \text{constant},$$

where m represents the photographic magnitude.

As the result of an elaborate experimental investigation, it was found that the exponent α has the same value, viz. 0·25, for plates of different emulsion, so that equation (3) assumes the definite numerical form

$$\log f(\mathrm{D}) = \log t - \tfrac{1}{2}m + \text{constant} \quad . \quad . \quad . \quad (4)$$

Suppose now that a number of exposures of different duration t_1, t_2, . . . of the same region of the sky are made successively on the same plate. Then for any given star whose magnitude may be called m_c relation (4) takes the form

$$\log f(\mathrm{D}) = \log t - \tfrac{1}{2}m_c + \text{constant} \quad . \quad . \quad . \quad (5)$$

Let D_1, D_2, . . . be the measured diameters of any given star corresponding with the times of exposure t_1, t_2, . . . Then the relation between D and $\log t$ can be graphically obtained by plotting D_1, D_2, . . . as ordinates against $\log t_1$, $\log t_2$, . . . as abscissæ. Every star supplies a graph of this character, and equation (5) shows that these graphs are similar and may be brought into coincidence by appropriate shifts along the $\log t$ co-ordinate. In this manner a general graph (D, $\log t$) may be constructed extending over the whole range of diameters. A curve of this kind will be denoted as the *time-scale* of the plate.

On the other hand, for the same time of exposure, t_c, but varying magnitudes m, the general equation (4) takes the form

$$\log f(\mathrm{D}) = \log t_c - \tfrac{1}{2}m + \text{constant} \quad . \quad . \quad . \quad (6)$$

Comparing equal diameters under the conditions (5) and (6), we obtain the relation

$$m - m_c = 2 \log t_c/t \quad . \quad . \quad . \quad . \quad (7)$$

where the term on the right-hand side can be obtained by ascertaining $\log t$ from the time-scale through the diameter D observed with exposure t_c. Hence the magnitudes of all the stars of the plate are known, provided that m_c can be determined from (7) for at least one star whose magnitude m is known from other sources.

The method of determining magnitudes by means of the relation (7) is well known, but has proved unsatisfactory in past attempts, since its limitation to stars whose incident light-intensity is much fainter than the optimal intensity of the plate had not been recognised.

On account of this limitation it is obvious that for a plate whose optimal intensity I_0 lies closely to the light-intensities of the stars, no accurate results can be expected from equation (7). For the fast plates used in astrographic work I found that the optimal intensity corresponds with the magnitude 3·0 approximately. The above formulæ show that for stars of this magnitude the relation (1) would have the form

$$f(D) = It,$$

and hence the factor in (7) would be 2·5 instead of 2. The applicability of equation (7) is in this case limited to stars which are at least 7^m fainter than the optimal magnitude, *i.e.* fainter than 10^m.

For slow plates, according to Abney and Kron, the optimal intensity is much greater, some 5 or 6 magnitudes brighter than for fast plates. Hence on such a plate equation (7) would be applicable already to stars of the 4th magnitude and fainter.

It is this difference in the behaviour of slow and fast plates on which the proposed method is founded. A selected region of the sky is photographed first on a slow plate. [The plate used for the purpose with the astrographic refractor was of the brand "Imperial Lantern." With one hour's exposure it shows measurable star-images down to about the 11th magnitude.] The time-scale (D, $\log t$) is constructed in the manner described above,* and the values $2 \log t_c/t$ are obtained for all the stars on the list showing measurable diameters. Next the same programme is performed on a plate of higher sensitiveness, the faintest stars of the slow plate being now the brightest measured objects on the faster plate. From the stars common to both plates the mean difference in the values of $2 \log t_c/t$ is ascertained, and this difference is applied to all the values of the second series, so as to refer them to the zero-point m_c of the first series. Thus by repetitions of the same programme on successively faster plates we eventually obtain values of $(m - m_c)$, referred to the *same* zero-point m_c, for a sequence extending from the brightest stars to the faintest objects within the optical grasp of the instrument.

The accuracy of the method can be readily tested for regions in which a sequence of photographic magnitudes has already been established on correct photometric principles. One of these, accessible in this latitude, is the group of the Pleiades, for which we possess a photographic star-sequence extending to the 15th magnitude, observed at Harvard, and published in *Harvard Annals*, 71, No. 3, table xlvi., and also a less extensive sequence to the 9th magnitude, observed

* An example of its construction is given at the end of the paper.

by Hertzsprung and published in *A.N.*, 4452. Both sequences may serve as standards of comparison. To present the material in a concise form, groups containing 3 or 4 stars were formed in order of ascending magnitude. They are denoted by the Roman figures I.–VIII. Table I. shows for each group the following data; (1) the mean photographic magnitudes according to Harvard (column 2); (2) the same according to Hertzsprung (column 3); (3) the values of $2 \log t_c/t$ found from six slow plates of the brand Imperial Lantern (column 4); (4) the values of $2 \log t_c/t$ found from two fast plates Ilford Monarch (column 5). Adding $m_c = 3{\cdot}56$ to the figures in columns 4 and 5, the photographic magnitudes of column 6 are obtained in accordance with the proposed method, their agreement with Harvard and Hertzsprung being shown in the differences of columns 7 and 8.

TABLE I.

Group.	m_p.		$2 \log t_c/t$.		m_p Cape.	Har. – C.	He. – C.
	Harvard.	Hertzsprung.	Slow.	Fast.			
I.	3·52	3·75	−0·13	...	3·43	+0·09	+0·32
II.	4·67	4·95	+1·12	...	4·68	−0·01	+0·27
III.	6·47	6·74	+2·98	...	6·54	−0·07	+0·20
IV.	7·55	7·85	+3·97	...	7·53	+0·02	+0·32
V.	8·86	9·22	+5·38	...	8·94	−0·08	+0·28
VI.	9·82	...	+6·29	+6·29	9·85	−0·03	...
VII.	11·15	...	...	+7·52	11·08	+0·07	...
VIII.	13·12	...	...	+9·53	13·09	+0·03	...

It will be seen that the scale of the Cape magnitudes is in good agreement with that of Harvard and Hertzsprung. To show, however, that this ceases to be the case for the bright stars on a fast plate, the following results, derived from two Ilford Monarch plates, may be quoted:—

TABLE II.

Group.	m_p (Hertzsprung).	$2 \log t_c/t$.	$(m_p - 2 \log t_c/t.)$
A	3·88	0·00	3·88
B	5·36	1·12	4·24
C	6·64	2·10	4·54
D	7·14	2·53	4·61
E	7·65	2·89	4·76
F	8·05	3·26	4·79
G	8·53	3·70	4·83
H	8·96	4·14	4·82
I	9·33	4·48	4·85

The last vertical column shows clearly that equation (7), on which the proposed method is founded, is not applicable to the brighter stars. There is a very pronounced systematic run in these differences, which approach asymptotically a constant value when the fainter groups are reached, thus showing that for stars fainter than group I. equation (7) becomes valid—a result in conformity with that of Table I.

As a further test, the attempt was made to derive photographic magnitudes by the proposed method for a star sequence in one of Kapteyn's selected areas recently published in *Harvard Annals*, **101.** For this purpose the equatorial area No. 101, R.A. $9^h\ 52^m$, was chosen. As in the case of the Pleiades, the results have been condensed by the formation of groups arranged in order of magnitude. Table III. shows these groups and the number of stars contained in each in columns 1 and 2. The magnitudes of column 3 are those published in the *Annals*, but reduced to the system of the Mt. Wilson north polar sequence by adding the following corrections depending on magnitude:—

m.	(Mt. W. – Harvard.)	*m.*	(Mt. W. – Harvard.)	*m.*	(Mt. W. – Harvard).
8·0	+0·07	9·5	+0·16	11·0	+0·22
8·5	+0·10	10·0	+0·19	11·5	+0·23
9·0	+0·13	10·5	+0·21	12·0 and >	+0·23

The region was exposed on three plates: a slow plate of the brand Imperial Lantern, a plate of medium sensitiveness of the brand Ilford Monarch, and a very fast plate of the same trade-mark. The values of $2 \log t_c/t$ or $(m - m_c)$ derived from these plates and brought into agreement at the overlapping points by additive constants are shown in columns 4, 5, and 6. Utilising the Mt. Wilson magnitude of the first group, we find $m_c = 9{\cdot}08 - 1{\cdot}22 = 7{\cdot}86$, and this figure, added to the quantities in columns 4, 5, and 6, leads to the photographic magnitudes shown in column 7, whose close agreement with the Mt. Wilson system throughout the whole range is demonstrated by the smallness of the residuals of column 8.

TABLE III.

Group.	No. of Stars.	m_p (Mt. Wilson).	$2 \log t_c/t$. Slow.	Medium.	Fast.	m_p (Cape).	Mt. W. minus Cape.
I.	6	9·08	1·22	...	...	9·08	0·00
II.	10	10·55	2·65	2·65	...	10·51	+0·04
III.	13	11·52	...	3·70	3·72	11·57	−0·05
IV.	8	12·17	...	4·32	4·30	12·17	0·00
V.	12	13·18	...	5·35	5·36	13·22	−0·04
VI.	8	13·87	...	...	5·94	13·80	+0·07
VII.	8	14·70	...	...	6·79	14·65	+0·05

The construction of the time-scale may be illustrated by an example. On the fast Ilford Monarch plate six exposures were given, the logarithms of the time-intervals expressed in seconds being

$$3{\cdot}556;\quad 3{\cdot}255;\quad 2{\cdot}954;\quad 2{\cdot}653;\quad 2{\cdot}352;\quad 2{\cdot}051.$$

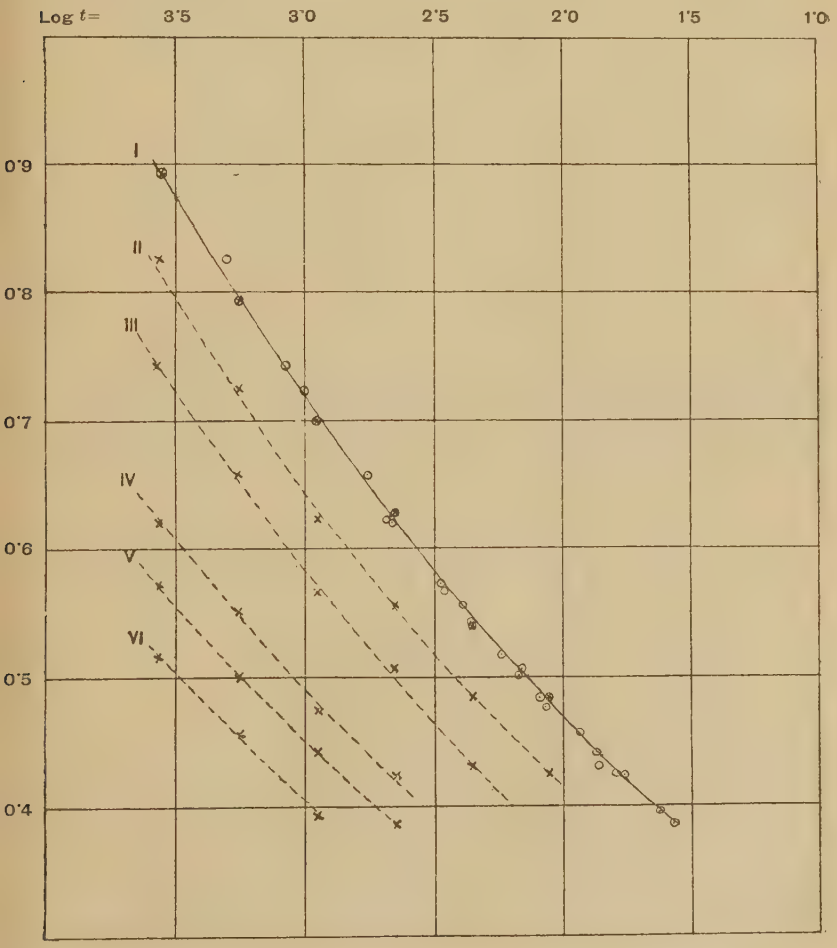
The observed diameters of the stars, expressed in revolutions of the micrometer screw and collected into six groups in order of brightness, were as follows:—

TABLE IV.

Measured Diameters.

	log t_c=	·3·556.	3·255.	2·954.	2·653.	2·352.	2·051.	No. of Stars.
Group	I.	0·891	0·793	0·700	0·629	0·540	0·485	5
	II.	0·826	0·725	0·622	0·557	0·484	0·425	4
	III.	0·743	0·659	0·567	0·507	0·430	...	6
	IV.	0·620	0·551	0·476	0·422	...	...	6
	V.	0·572	0·500	0·441	0·386	...	...	4
	VI.	0·518	0·458	0·394	...	...	...	5

Plotting for any of the groups the diameters as ordinates against the corresponding log t_c as abscissæ and connecting the end-points of the ordinates (marked by crosses in the adjoined diagram) the dotted curves



are constructed. Each of these may be selected as the time-scale of the plate. By applying the following corrections to the values of $\log t_c$

Group	I.	0·00	Group	IV.	−0·89
	II.	−0·26		V.	−1·08
	III.	−0·49		VI.	−1·32

and accepting the corrected values as abscissæ, the crosses in the diagram are shifted into the positions of the small circles which group themselves satisfactorily along the drawn-out curve. The correlated co-ordinates of this curve represent the finally adopted time-scale shown in Table V.

TABLE V.

Diameter.	log *t*.
0·9	3·58
0·8	3·26
0·7	2·93
0·60	2·56
0·55	2·36
0·50	2·14
0·45	1·91
0·40	1·64

To find now the values of $(m - m_c)$ or $2 \log t_c/t$ for the groups I.–VI. (and in the same manner for every individual star), we enter Table V. with the observed diameter of Table IV., take out the corresponding value of log *t*, deduct it from the log t_c heading the same column, and multiply the difference by 2. Thus we obtain :

		$m - m_c$.						Mean $(m - m_c)$.
Group	I.	+0·01	+0·03	+0·05	−0·04	+0·06	−0·04	+0·02
	II.	+0·43	+0·49	+0·62	+0·52	+0·56	+0·54	+0·53
	III.	+0·97	+0·93	+1·05	+0·96	+1·08	...	+1·00
	IV.	+1·83	+1·79	+1·86	+1·78	...	...	+1·81
	V.	+2·20	+2·23	+2·19	+2·18	...	...	+2·20
	VI.	+2·67	+2·61	+2·69	...	...	...	+2·66

On the Rotation of the Sun's Reversing Layer.

By J. Halm.

Professor Newall's paper "On the Law of Rotation of the Sun" in *M.N.*, 82, 101, has raised once more the much-debated question of a variability of the sun's rotation during a cycle of solar activity. My Edinburgh observations made during the years 1901–6, *i.e.* during a time of change of solar activity from a minimum in 1901 to a maximum in 1905–6, have revealed changes in the annual means of the observed velocities which were considered far too large and systematic to be attributable to errors of an instrumental or personal character.

It was pointed out at the time (*A.N.*, 4147, p. 295) that a strong evidence against the assumption of such large instrumental errors is to be found in the fact that the Edinburgh series for 1901 is in close systematic agreement with Dunér's observations of the same year. On the strength of this agreement I ventured to suggest that the observed variations might be due to actual changes in the surface rotation of the sun depending on the state of solar activity. A definite opinion on this phenomenon had naturally to be deferred until at least one complete cycle had been covered by observations. The deciding question was: Would the conditions of solar rotation peculiar to 1901, a year of minimum solar activity, be repeated at the subsequent minimum in 1911–12?

A paper dealing with a discussion of all the spectroscopic observations of solar rotation up to 1913 had already been prepared some years ago, but its publication was deferred, as it was hoped that further observations might be forthcoming comprising the epoch of maximum activity 1915–16. Unfortunately no records of such observations are available up to the present moment. In view of the possible bearing which the results of the present communication may have on the interesting theoretical views expressed in Professor Newall's paper, it is thought advisable to publish the paper at this juncture.

The observational material so far available is as follows:—

Upsala observations	(Dunér),	1887–89 and 1899–1901 ; *A.N.*, 3994.
Edinburgh	,,	(Halm), 1901–6 ; *A.N.*, 4147.
Mt. Wilson	,,	(Adams), 1906–7 and 1908 ; *Ap. J.*, 26, 29.
Edinburgh	,,	(Storey and Wilson), 1909 ; *M.N.*, 71.
Ottawa	,,	(Plaskett and De Lury), 1911, 12, 13 ; *Ap. J.*, 42.

The following procedure of discussion of these observations was adopted. For each annual series the observed mean values of the linear velocities, v, as recorded by the observers were plotted as ordinates against the corresponding heliographic latitudes, ϕ, as abscissæ, and a smooth curve was drawn as closely as possible through the end-points of the ordinates. From the graphs thus obtained the values of v for every 5th degree of latitude were read off and entered in the annexed table. The curves represent the observed values so closely that the table may be considered to give a faithful record of the linear velocities

of the reversing layer at the specified latitudes as observed at the epochs noted in the headings.

As regards the state of solar activity at the specified epochs, all the Upsala records refer to times of *minimum* activity. The Edinburgh series begins at a sunspot minimum (1901) and extends beyond a sunspot maximum (1905–6). The Mt. Wilson series begins in 1906 and is continued to the end of 1908. During this period solar activity was declining; the observations may be considered to represent a state of solar activity intermediate between maximum and minimum. Finally, the Ottawa observations 1911–13 were made at a time of minimum solar activity, and therefore should represent conditions of solar rotation similar to those of the Upsala observations.

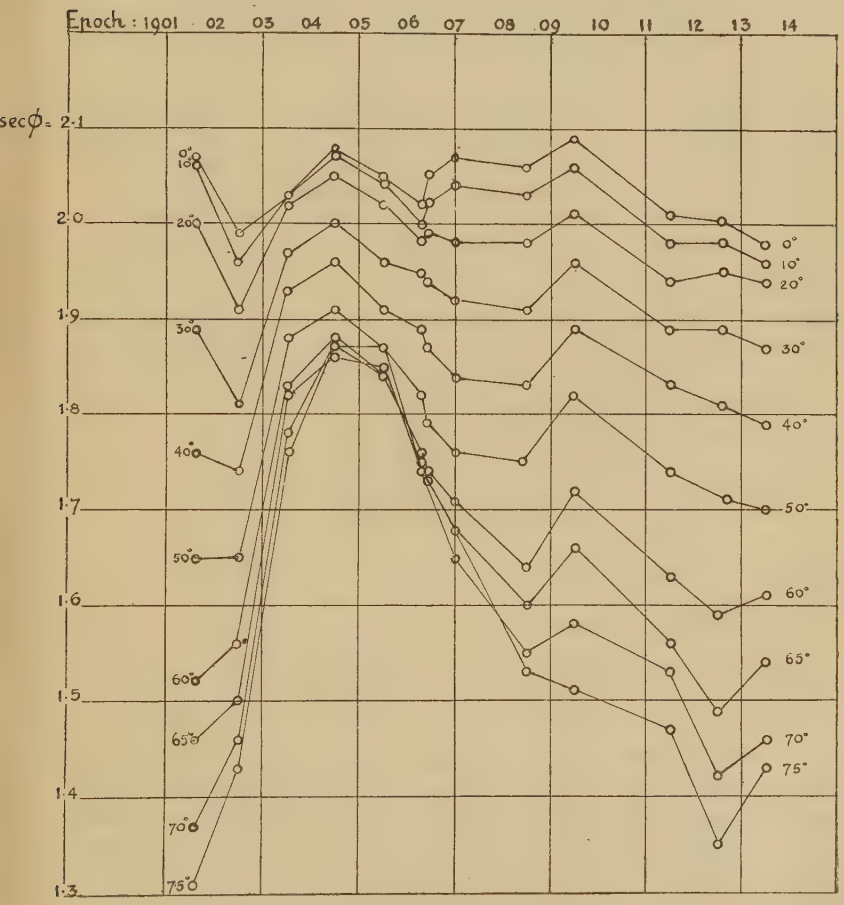
Hence three epochs of minimum solar activity are covered by the Upsala, Edinburgh, and Ottawa observations, whereas only one sunspot maximum is covered by the Edinburgh observations 1904–6. The answer to the question as to whether the surface rotation of the sun is subject to variations depending on the sunspot cycle will thus largely depend on the Edinburgh observations. It is highly important, therefore, to test their reliability by comparison with adjoining series. For this purpose the Upsala observations of 1901, whose epoch agrees very closely with that of the Edinburgh observations of the same year, are shown separately (see table, columns 4 and 5). The figures leave no doubt as to the systematic agreement between the results obtained at the two observatories Upsala and Edinburgh at the same epoch.

The last Edinburgh series 1906·3 is followed by the Mt. Wilson series 1907·0, which comprises observations extending over the period 1906 May 3 to 1907 June 23. If, however, the earliest eleven plates are considered by themselves, the set of velocities in the column (Mt. W., 1906·4) is obtained which differ in time from the Edinburgh set only by 0·1 of a year. The systematic agreement between Edinburgh and Mt. Wilson for this common epoch is remarkably close. We have thus, on the one hand, agreement between Upsala and Edinburgh, and, on the other, between Edinburgh and Mt. Wilson, a fact which doubtless vindicates the Edinburgh observations against the suspicion of being vitiated by systematic errors of an instrumental or personal character. The agreement between Edinburgh and Mt. Wilson is the more remarkable as the Edinburgh observations are based on two iron lines only, whereas the Mt. Wilson measurements comprise a considerable number of lines characteristic of various chemical elements. Adams' table on p. 109 of his work on *An Investigation of the Rotation Period of the Sun by Spectroscopic Methods* shows the deviations of angular velocities for individual lines from the mean value. According to the published figures, the iron lines show the following differences in the linear velocities:—

At 0° + 0·002; at 30° + 0·003; at 60° + 0·002; at 80° + 0·001 km./s.

The differences are quite negligible, and thus demonstrate that the homogeneity of the material embodied in the preceding table is in no way impaired by the different selection of lines. This conclusion, however, applies strictly only to lines originating in the level of the

reversing layer. Adams found notable differences in the case of hydrogen and calcium. As an example, the linear velocities of rotation as measured by the H_α-line inside the limb are shown in the last vertical column. Comparison with the reversing layer observed at the same epoch (1908·5) shows most appreciable differences, amounting *in maximo*



to 0·12 km./sec. It is worthy of notice, however, that the H_α rotation of 1908 agrees almost exactly with the rotation of the reversing layer in 1904.

Having now at our disposal observational material extending almost uninterruptedly over a complete cycle of solar activity from 1901 to 1913, and having demonstrated that the results of the three observatories which have contributed the greatest share to the work, viz. Upsala, Edinburgh, and Mt. Wilson, are in satisfactory agreement with

one another, we proceed to examine what information can be obtained regarding the constancy or variability of the period of rotation of the reversing layer.

Evidently, if the layer in question rotated like a solid spherical shell, the following result should be anticipated. Multiplying the linear velocities of the table by the factor $\sec\phi$, and plotting the values of $v \sec\phi$ as ordinates against the epochs of observation as abscissæ, the locus of the end-points should be a horizontal line, which is obviously the same whatever latitude be selected for examination.

If, on the other hand, the angular velocities $v \sec\phi$ depend on the latitude, but not on the time, then, plotting them in the manner indicated, we should obtain a separate horizontal line for every latitude. Systematic departures from horizontality would mark changes in the angular rotation depending on the time of observation.

The graphical results are shown in the accompanying diagram. The departures from horizontality, more particularly in the higher latitudes, show emphatically that the results are incompatible with the view of a constant law of rotation. Even if the observations prior to 1906 were disregarded, the downward slope of the high-latitude curves from 1906 to 1913 militates most strongly against the assumption of an invariable solar rotation. The general evidence of the observations leads to the conclusion that *the same law of rotation of the reversing layer can be expected only under similar conditions of activity.*

Linear Velocities of Rotation at Different Heliographic Latitudes and at Different Epochs of Observation.

Heliographic Latitude ϕ.	1887-89 Ups.	1899-1901. Ups.	1901·5. Ups.	1901·7. Ed.	1902·5. Ed.	1903·5. Ed.	1904·5. Ed.	1905·5. Ed.	1906·3. Ed.
°	km./s.								
0	2·09	2·03	2·07	2·08	1·99	2·03	2·08	2·05	2·03
5	2 07	2·02	2·06	2·06	1·96	2·02	2·07	2·03	2·02
10	2·02	1·99	2·02	2·02	1·93	1·99	2·04	2·01	1·98
15	1·96	1·94	1·96	1·95	1·86	1·95	2·00	1·97	1·92
20	1·88	1·87	1·88	1·86	1·79	1·89	1·93	1·89	1·86
25	1·80	1·79	1·78	1·75	1·68	1·81	1·83	1·80	1·78
30	1·69	1·68	1·65	1·62	1·57	1·71	1·73	1·70	1·69
35	1·55	1·54	1·51	1·49	1·45	1·59	1·61	1·58	1·58
40	1·41	1·40	1·36	1·36	1·33	1·48	1·50	1·46	1·44
45	1·26	1·25	1·20	1·22	1·20	1·35	1·37	1·34	1·30
50	1·11	1·10	1·05	1·07	1·06	1·21	1·22	1·20	1·16
55	0·97	0·94	0·89	0·92	0·92	1·06	1·08	1·06	1·02
60	0·81	0·80	0·75	0·76	0·78	0·91	0·94	0·92	0·88
65	0·67	0·66	0·60	0·61	0·63	0·76	0·78	0·78	0·73
70	0·53	0·52	0·46	0·47	0·50	0·61	0·63	0·63	0·60
75	0·40	0·38	0·33	0·34	0·37	0·45	0·48	0·48	0·45
80	0·26	0·25	0·21	0·22	0·24	0·30	0·34	0·35	0·30

Linear Velocities of Rotation at Different Heliographic Latitudes and at Different Epochs of Observation—continued.

Heliographic Latitude ϕ.	1906·4. Mt. W.	1907·0. Mt. W.	1908·5. Mt. W.	1909·5. Ed.	1911·5. Ottawa.	1912·5. Ottawa.	1913·5. Ottawa.	Mean, 1901–13.	H_α, 1908·5. Mt. W.
°	km./s.								
0	2·05	2·07	2·06	2·09	2·01	2·00	1·98	2·038	2·11
5	2·03	2·04	2·04	2·07	1·98	1·98	1·96	2·019	2·09
10	1·99	2·00	2·00	2·02	1·95	1·95	1·93	1·987	2·05
15	1·94	1·94	1·94	1·96	1·89	1·89	1·88	1·933	2·00
20	1·87	1·86	1·86	1·89	1·82	1·83	1·82	1·861	1·93
25	1·78	1·77	1·76	1·80	1·74	1·75	1·74	1·770	1·83
30	1·68	1·66	1·65	1·70	1·64	1·64	1·62	1·662	1·73
35	1·57	1·54	1·54	1·58	1·53	1·52	1·50	1·544	1·61
40	1·43	1·41	1·40	1·45	1·40	1·39	1·37	1·417	1·50
45	1·29	1·27	1·27	1·31	1·26	1·24	1·23	1·281	1·37
50	1·15	1·13	1·12	1·17	1·11	1·10	1·09	1·138	1·22
55	1·01	0·99	0·97	1·02	0·96	0·95	0·95	0·994	1·08
60	0·87	0·85	0·81	0·86	0·81	0·79	0·80	0·845	0·93
65	0·73	0·71	0·67	0·70	0·67	0·63	0·65	0·696	0·78
70	0·59	0·56	0·53	0·54	0·52	0·48	0·50	0·549	0·63
75	0·45	0·43	0·39	0·39	0·38	0·35	0·37	0·408	0·48
80	0·30	0·28	0·26	0·25	0·24	0·22	0·24	0·272	0·34

Mean Areas and Heliographic Latitudes of Sun-spots in the Year 1918.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those given in the *Monthly Notices*, 82, 62, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at Kodaikánal Observatory and at Dehra Dûn.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun for the year 1918, and Table II. gives the same particulars for each entire year from 1913 to 1918 inclusive. The areas are given in two forms: firstly, projected areas, *i.e.* as seen and measured on the photographs, these being expressed in millionths of the Sun's apparent disc; secondly, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. gives for each rotation in the year 1918 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for spots north and for spots

south of the equator, together with the mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots. Table IV. gives the same information for each year as a whole from 1913 to 1918. Tables II. and IV. are thus in continuation of similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*; for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1914 on pp. 402 and 403 of vol. lxxvi.

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

The principal features of the record for the year 1918 are as follows:—

1. The marked fall in the mean daily area from 1537 (corrected for foreshortening) for the year 1917 to 1118 for the year 1918. The maximum of this cycle is thus sharply defined, being similar in this respect to the maximum year of 1893 and contrasting with the double peak of the maximum of the last cycle in the years 1905 and 1907.

2. The fluctuations in the mean daily area for the separate rotations are considerable for the first half of the year. A maximum is shown in rotations 859 and 860, and a secondary but more sustained maximum in rotation 868.

3. The mean daily area of faculæ shows a decrease similar to that of the spots, but the variations in the amount from rotation to rotation are smaller. The second half of the year shows a preponderance.

4. Comparing the two hemispheres, the greater activity of the northern is continued. Of the entire spotted area, 54 per cent. occurred in the northern hemisphere.

5. The equatorwards progression of the mean latitude of spots is consistently maintained during 1918.

6. On no day in the year 1918 was the Sun's disc free from spots.

7. The number of separate groups of spots observed during 1918 for at least two days was 404 as compared with 455 for 1917. Of these 404 groups, 205 were in the northern hemisphere and 199 in the southern. Besides these groups there were recorded 192 small groups (mostly consisting of one or of two very small spots) seen for one day only. These were divided between the N. and S. hemispheres in the proportion of 102 to 90 respectively.

Royal Observatory, Greenwich:
1922 *June.*

TABLE I.

No. of Rotation.	Rotation commenced G.M.T. (Civil).		No. of Days on which Sun photographed.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
	1918	d							
860	Jan.	3·41	27	345	2393	1417	255	1821	1753
861	Jan.	30·75	27	171	1118	1308	122	821	1608
862	Feb.	27·09	27	186	1091	1047	138	835	1314
863	Mar.	26·41	28	242	1336	1254	171	952	1573
864	Apr.	22·68	27	251	1372	1530	193	1058	1876
865	May	19·91	27	224	1134	1419	162	818	1733
866	June	16·12	27	187	1056	1264	126	720	1616
867	July	13·32	28	339	1876	1642	256	1453	2039
868	Aug.	9·53	27	331	2025	1833	248	1518	2231
869	Sept.	5·77	27	246	1488	1625	179	1088	1992
870	Oct.	3·05	27	293	1680	1916	210	1231	2331
871	Oct.	30·34	27	257	1443	1836	187	1046	2255
872	Nov.	26·65	27	223	1400	1643	169	1037	2015

TABLE II.

Year.	No. of Days on which Sun photographed.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
1913	365	2	10	87	1	7	95
1914	365	40	195	396	31	152	454
1915	365	186	920	1289	137	697	1521
1916	365	186	978	1512	135	724	1785
1917	365	346	2100	1882	248	1537	2305
1918	364	255	1504	1526	188	1118	1882

TABLE III.

No. of Rotation.	Rotation commenced G.M.T. (Civil).		Spots NORTH of the Equator. Mean of Daily Areas.	Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean of Daily Areas.	Mean Heliographic Latitude.	Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
	1918	d		°		°	°	°
860	Jan.	3·41	808	11·58	1013	15·20	−3·31	13·59
861	Jan.	30·75	600	6·28	221	16·70	+0·09	9·09
862	Feb.	27·09	624	14·39	211	14·61	+7·05	14·44
863	Mar.	26·41	558	13·79	394	11·45	+3·35	12·82
864	Apr.	22·68	612	16·19	446	13·63	+3·62	15·11
865	May	19·91	342	14·64	476	13·35	−1·64	13·89
866	June	16·12	516	11·35	203	13·40	+4·36	11·93
867	July	13·32	863	11·89	590	16·05	+0·53	13·56
868	Aug.	9·53	1039	11·74	479	16·64	+2·79	13·29
869	Sept.	5·77	470	11·72	618	12·38	−1·97	12·10
870	Oct.	3·05	882	11·46	349	14·64	+4·06	12·36
871	Oct.	30·34	624	11·36	422	13·15	+1·48	12·08
872	Nov.	26·65	58	12·98	979	11·03	−9·69	11·14

TABLE IV.

Year.	No. of Days on which Sun photographed.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	d		°		°	°	°
1913	365	5	24·81	2·5	20·08	+9·81	23·23
1914	365	99	21·11	53	23·06	+5·71	21·79
1915	365	379	17·95	318	19·75	+0·77	18·77
1916	365	470	15·14	254	17·05	+3·83	15·81
1917	365	860	13·82	677	15·66	+0·84	14·63
1918	364	609	11·97	509	13·69	+0·29	12·75

Notes on Four Interesting Nebulæ. By C. D. Perrine.

N.G.C. 1291 $3^h\ 13^m\ 43^s$ $-41°\ 27'·9$

This nebula presents the unusual feature for a spiral of being surrounded by a large apparently disconnected ring. It is described in the *N.G.C.* as "⊕, vB, pL, R, mbM, er." Harvard observers (*Annals H.C.O.*, **76**, No. 4, p. 45) describe this object from photographs with the Bruce telescope as follows "B, hazy ✲, vB at M, r, alm R." Three photographs secured with the 75-cm. reflector with exposures of one hour each show that it is not a globular cluster, but is probably a spiral nebula of exceptional interest.

The central bright part is approximately 1′ in diameter and without detail, looking like a hazy ball. Outside this are elongations or arms of very faint nebulosity nearly north and south, the diameter in that direction being 3′. This nebulosity also extends outward at right angles to the longer axis a short distance beyond the ball, giving it an elliptical form, or more probably a spiral or ring formation seen at an angle. This part of the object resembles a type which is common among the spirals of medium size. Between this central structure and the outer ring is a space about 1′ wide which appears to be devoid of nebulosity.

When examined carefully the large outer ring is seen to be composed of about one and one half turns of a helix, the doubled portion (showing two streams) being to the north of the nuclear portion of the nebula. The diameter of the helix is about 7′. The diameter or perhaps major axis at right angles to the major axis of the central portion is 10′. The structure in these parts where the overlapping turns end is not well defined and gives the appearance of ellipticity in a generally east-west direction. This outer ring is very faint, but the general features are distinguishable in all the negatives. The inner edge of the ring is not much larger than the diffraction rings about the brighter stars, but is

less sharp. The general appearance and its larger size as well as much fainter nucleus seemed to preclude any possibility of its being spurious. The star *G.C.* 3577, magnitude $7\frac{1}{4}$, is about 10′ south-west of the nebula. The one-hour exposures show a very faint diffraction ring about this star on the unbacked plates. To settle definitely such a possibility, a third plate was taken, backed with black paper and glycerine. No ring is shown about the star on this plate, but the ring about the nebula is unchanged. The apparently vacant space between the central portion and the outer ring may be non-luminous matter, or it may contain some matter, but too sparsely distributed to show on these negatives. That it is non-luminous matter seems doubtful because of its relatively great extent, lack of the sharply defined edges and structure which usually characterise the streams of such matter in other spirals, and also from the helical structure of the outer ring, a structure which has generally been associated with streams of matter. Much stress is not laid on analogy, however, in such a case.

Enlarged copies on Seed 23 plates show the space between the central mass and the outer ring to be darker than the sky background. The original negative does not show this, even when examined at a small inclination to the sight-line to increase the contrasts. More extensive investigation will be necessary to decide whether this apparent darkening is the legitimate outcome of the well-known effect of increasing contrasts by copying, or whether it is simply physiological. If a real darkening is shown, it would point to non-luminous matter instead of vacancy.

In my experience, an external ring or helical formation about a spiral, such as this appears to be, is unique.

N.G.C. 4027	$11^h\ 54^m\ 25^s$	$-18°\ 43'$
4038–9	11 56 46	−18 19

The internal motions of the spiral nebulæ, on account of the prevalence of these objects and this type of motion, furnish one of the most interesting problems of celestial mechanics.

Any evidence which will throw light on the problem becomes of importance, and it is with that thought in mind that I call attention to the three objects, noted above as separate nebulæ, in the *N.G.C.* 4038–9 are but parts of the same object. A plate containing all of them was obtained with the 75-cm. reflector on 1918 March 11, with an exposure of one hour. These nebulæ are small, and, except for their peculiar forms and possible relation, would call for little notice. 4027 is listed in the *N.G.C.* as a globular cluster, which is clearly erroneous as noted by the Harvard observers. Calling 4038–9 one object, these two nebulæ are about 42′ apart, the line joining them being at approximately 55° to the meridian, and roughly making the same angle with the galaxy.

There are several peculiarities in these nebulæ.

1*st. General Appearance.*—This is such as to classify them together, they being both differentiated from the average small spiral. They are neither of them perfect spirals, yet together they make one. They are similar in general appearance.

2*nd. Form.*—Each may be described as a hook extending out from a ring, these hooks resembling each the arm of a spiral. No indications of second opposite arms can be seen in either.

3*rd. Structure.*—They appear to be largely nebulous with several bright knots on the (slightly) larger and one such knot at the origin of the smaller one. In this matter of texture they are both different from most spirals and appear remarkably alike.

The star images are rather large due to poor seeing and guiding. This, however, cannot change the essential conclusions above as to structure.

4*th. Position Angles of Arms.*—The directions of the arms are approximately at right angles to the line joining the two nebulæ, on opposite sides. If brought together along the line joining them, the arms would be on opposite sides of the central rings. By rotating one with respect to the other 50° or 60° in position angle, and superposing the central rings, an excellent spiral is formed.

5*th. Curvature of Arms.*—These are opposite as they appear in the sky—similar to that of the spiral nebulæ. Considering the curvatures as related to the central mass and the motion as outward, they are both clockwise.

6*th. Size.*—The over-all length of the best defined portion of each arm is approximately the same, 1′·5. The central rings are not perfect circles, but are somewhat irregular and are flattened slightly, their greatest diameters being about $\frac{2}{3}'$ each. If these hooks are superposed, the similarity in both size and general appearance is striking.

There are several very small elliptical and round nebulæ on the plate, but no others anything like the size or appearance of these. Nothing abnormal is to be seen in the distribution of the stars, or their number or brightness. The region appears to be normal for spiral nebulæ at least. The considerable distance from the Milky Way also points to the region of spirals and not to irregular gaseous or planetary nebulæ. That there is some relation to spiral motion seems fairly evident. Whether the two objects are closely related one to the other or are sporadic formations, their nearness being purely accidental, is speculative. The similarity in size, texture, and form; the complementary curvature and directions of the arms; the lack of the secondary arm in both, make it difficult to believe that there is not some intimate relation. The doctrine of probability would certainly be outraged otherwise.

In my experience with the nebulæ of both hemispheres, these two are unique. In one or two cases I recall irregular structure such that one of the arms was fainter or less definitely developed, but none in which only one well-defined arm could be discerned. In essentially all of these hundreds of spirals, two opposite, equally or nearly equally well-defined arms, similar in appearance and extension, are plainly observable; so much so that observation bears out the conclusion of dynamics regarding the formation of two opposite arms in normal cases. It is perhaps well to remember in this connection the general prevalence of and belief in obscuring dark matter in the spiral nebulæ. No traces of such matter can be found, however, in or about these nebulæ. At

least, not such as could conceivably occult the missing arms. This and other facts throw doubt on the possibility of obscuration.

That two such objects should have had a similar or common origin seems highly speculative, and yet the facts at present point to such a possibility. If the apparently complementary relations are pure accident, it is a most remarkable case of coincidence. Observations of their relative angular motions, radial velocities, etc., which it is hoped may be secured in the not distant future, should throw light on the matter. As these objects are within the reach of several of the great reflectors of the northern hemisphere, I am publishing this preliminary note in the hope that others will put them on their observing programs. If the material is largely gaseous it may be possible to obtain an impression of the spectrum.

In seeking for information that will help to explain these appearances, analogy furnishes but little aid. There is, however, one class of body which may have a bearing—the truly helical nebulæ, of which there are at least a few examples that appear to be unquestionable. Perhaps the more numerous θ type spirals may have a relation as they also possess central rings. Neither of these forms so far, however, has been found to possess but a single arm. The radial velocities of the spirals as a class have been observed by Slipher at Lowell Observatory to be high, the highest of any class of celestial bodies. The Mount Wilson observers find unexpectedly large outward displacements. Can it be that these outward motions and high velocities point to disruption and final separation in exceptional cases? or have the two bodies been formed under similar conditions but remote from each other? If this is the case, why are they unique in each having only one arm and to be apparently so complementary in several ways. Are they "sports" in which normal development was interfered with by some extraneous factor usually lacking? Are there single-arm spirals in which the matter is moving inward instead of outward?

N.G.C. 6822 $19^h\ 39^m$ $-15°\ 1'$
v F, L, E, dif.

This nebula, discovered by Barnard, has no counterpart in my experience among the smaller nebulæ, but resembles the Magellanic clouds. A negative made with the 75-cm. reflector on 1919 September 19, with an exposure of one hour, shows it to be 10′ long by 5′ wide, roughly elliptical, with the longer axis in a north and south direction, which in that region of the sky is nearly parallel to the galactic plane. Although close to the Milky Way the plate shows none of the characteristic cloud groupings among the stars, such as are so marked on similar exposures in the Milky Way proper. The background of stars on this plate resembles that of non-galactic regions, and the impression given is that this object is farther away than the background of stars.

The nebula is nearly but not quite uniform in structure, but presents no more evidence of spiral structure than do the Magellanic clouds, both of which are curved somewhat. Its appearance, and the fact that it is visually seen at all, indicates a middle and later type spectrum, for it is

a very faint object on the negative. Along one extremity are several very small apparently nebular objects, again in general resemblance to the larger clouds. Indeed, the appearance of the nebula on the plate is best described by saying that it looks like the small cloud to the unaided eye.

N.G.C. 6935–7	$20^h\ 30^m\ 57^s$	$-52^\circ\ 27'$
	20 31 23	52 30

While several well-defined types of spiral nebulæ are recognised, outside of the very small structureless ones, it is very rare indeed to find two near together which are alike. A negative having an exposure of one hour, and showing both the above objects well situated, was secured with the 30-inch reflector on 1919 October 15. They are so nearly alike in size and structure that, as they are so close together (5′), they call for remark. No others in my experience, with the exception of the pair of hooks (*N.G.C.* 4027, 4038–9), present such striking similarity. Indeed, these latter in the matter of general appearances are not so striking as 6935–7, which are almost exact counterparts except for brightness. They are both almost perfect circles of essentially the same diameter (40″–50″), and having central stars of the same magnitudes (15). The nebulosity of 6937 is considerably fainter, however, than that of the preceding one, 6935. No other nebulæ of appreciable size are within a degree. Their ring forms and central stars give them strongly the appearance of planetaries. There are suspicions, however, of very faint structures about them both, which may be spiral. This is most marked in the case of 6937, in which three spiral arms can be traced (two of them opposite). They are so faint, however, that the condensations of which they appear to be composed may be nothing more than accidental arrangements of very faint stars. The suspected structure about 6935 is even less certain. A spiral rather than planetary character is indicated also by their considerable galactic latitude. Further investigation, probably with larger telescopes, will be necessary to decide on the reality and form of these suspected outer features. Any further investigation at the present time is impossible, as even the 30-inch reflector is out of commission.

Córdoba:
1922 *March* 30.

Addendum. (1922 May 9.)

Since the above notes were prepared I have received Bulletin No. 21 of the Helwân Observatory and the annual report of the Mount Wilson Observatory for 1921. The former contains short descriptions of *N.G.C.* 1291 and 4027, 4038–9, and the latter of *N.G.C.* 4038–9 and 6822.

The Helwân description of 1291 agrees with Córdoba for the inner portion. The outer helical ring was evidently too weak to impress itself upon their plate at its great zenith distance, 71°. The Helwân descriptions of *N.G.C.* 4027 and 4038–9 are, for both, "single-branched

spirals," and note a similarity in appearance and opposite directions for the arms.

The Mount Wilson report describes 4038–9 as "a small bright spiral of extraordinary form." 4027 does not appear to have been observed by them. They describe 6822 as "a remarkable group of stars and small nebulæ, with probably some diffuse nebulosity."

These entirely independent observations and descriptions of 4027, 4038–9, and of 6822, confirm, in a very satisfactory manner, the Córdoba negatives, and strengthen my belief in the importance of these objects. The Mount Wilson observation of 6822 leaves little doubt in my mind that this object is in reality a third nubecula.

Note sur la précision des signaux horaires émis de 1910 *au* 31 *décembre* 1919 *par le Bureau International de l'Heure à Paris.* Par B. Baillaud.

On sait que, sur l'initiative du Bureau des Longitudes, l'émission des signaux horaires, par la Tour Eiffel et l'Observatoire de Paris, a commencé le 23 mai 1910 et a continué depuis sans interruption. Des conférences internationales tenues à l'Observatoire de Paris à partir du 15 octobre 1912 et à la fin d'octobre 1913, la seconde ayant un caractère non seulement technique mais aussi diplomatique, ont réglementé cet envoi. A la suite de la création à Bruxelles, en novembre 1919, de l'Union Internationale Astronomique, le service des signaux releva non plus du Bureau International de l'Heure créé en 1913, mais de la Commission de l'Union Internationale, commission comprenant M. Sampson, astronome-royal pour l'Ecosse, Président, M. B. Baillaud, le général Ferrié, M. G. Bigourdan, secrétaire. C'est au secrétaire qu'incombe la direction du nouveau Bureau. L'ancien avait pour président A. Backlund, directeur de l'Observatoire de Poulkovo; pour vice-président M. Lecointe, directeur de l'Observatoire d'Uccle-Bruxelles; pour secrétaire général, M. E. Kohlschütter, fonctionnaire du Ministère de la Marine à Berlin; M. B. Baillaud était Directeur du Bureau.

La détermination de l'heure, c'est-à-dire de la correction chaque jour, à heures fixées d'avance, d'une pendule fondamentale, dite pendule directrice, incombait au personnel de l'Observatoire de Paris. Les détails concernant cette organisation sont donnés, avec des recherches théoriques dans un rapport spécial. Des améliorations importantes ont été produites par l'introduction dans le travail (1°) en 1912 d'une pendule de Riefler installée dans nos caves à pression et température constante, don d'un astronome français très distingué; (2°) par l'emploi d'une pendule analogue 1185 Leroy établie le 12 juin 1913 dans une vitrine de la salle des pendules et maintenue à une température voisine de 21°. Les observations astronomiques de détermination de l'heure furent faites au Grand Cercle méridien, au Cercle méridien du Jardin, à l'instrument de longitudes de P. Gautier.

A certaines périodes, nos signaux furent reçus par divers observa-

toires et comparés à leurs propres déterminations de l'heure. Il convient que nous signalions tout spécialement ici les grands services que nous a rendus Sir Fr. Dyson, qui chaque jour depuis le 30 juillet 1912, nous a envoyé une carte postale contenant les heures de Greenwich auxquelles ont été reçus à Greenwich les signaux ordinaires du matin et souvent les résultats du soir. En rendant comparables ces résultats on trouve pour les différences G – P.

	Signaux ordinaires.		Signaux scientifiques.
	Matin.	Soir.	
1912	$+0^s\cdot04$	$+0^s\cdot05$	$+0^s\cdot11$
1913	$-0^s\cdot05$	$-0^s\cdot05$	$-0^s\cdot035$

On doit signaler ici que le temps sidéral à midi moyen de Paris employé dans les réductions de Paris est tiré de la *Connaissance des Temps* et supérieur de $0^s\cdot06$ à celui que donnent les autres éphémérides.

Pour les années suivantes on a pour les signaux du matin :—

	1914.	1915.	1916.	1917.	1918.	1919.
G – P	$-0^s\cdot07$	$-0^s\cdot02$	$-0^s\cdot10$	$-0^s\cdot05$	$-0^s\cdot04$	$0^s\cdot00$

A partir de juin 1915, M. Sampson nous a transmis très fréquemment des télégrammes donnant les comparaisons sous la forme Heure Ed.—Heure Paris. Rendus comparables par l'application de la correction $-0^s\cdot06$, les résultats moyens ont été :—

E – P	$-0^s\cdot12$	$-0^s\cdot08$	$-0^s\cdot11$	$-0^s\cdot15$	$-0^s\cdot14$

La discussion des comparaisons de pendules faites à Paris de 1914 à 1919, et celle de marches de pendules déduites des observations astronomiques de Paris, ont donné les résultats suivants de très grande importance pour le Bureau International de l'Heure, et aussi pour l'appréciation de la valeur des observations méridiennes.

Dans les périodes 1914–19 on a comparé par coïncidence, en passant par l'intermédiaire d'une pendule de temps moyen, les 2 pendules Riefler D. et 1185 Leroy. Des études patientes ont conduit à représenter pendant des périodes de 21 jours la différence des états de ces pendules par des formules paraboliques $a + bt + ct^2$, dans chacune desquelles l'origine du temps était la date moyenne de la période. Les écarts moyens entre les valeurs absolues des nombres observés, et les nombres calculés ont été en centièmes de seconde de temps :—

1914.	1915.	1916.	1917.	1918.	1919.	Moy.
1·7	0·9	1·0	1·0	1·2	1·4	1·2

La représentation des corrections de ces deux pendules déduites des observations astronomiques a donné pour la moyenne des valeurs absolues des écarts :—

Riefler	3·4	2·3	2·2	3·1	2·5	2·6	2·7
1185 Leroy	3·0	2·2	2·7	2·9	2·7	3·2	2·8

Il suit delà quel l'on peut admettre qu'en dehors de cas exceptionnels la marche de chacune des pendules peut être représentée par une formule parabolique au centième de seconde près; les écarts plus forts donnés par la représentation des résultats des observations méridiennes correspondent aux erreurs de ces observations.

Les heures d'émission de nos signaux chaque jour résultent le plus souvent d'extrapolation. Les corrections à leur appliquer ont eu pour moyenne de leurs valeurs absolues:—

7·3 2·9 4·1 4·8 5·4 5·3 5·0

La conférence de 1913 avait demandé que les signaux horaires fussent exacts à $\frac{1}{3}$ de seconde près. Ils l'ont été en moyenne à un vingtième de seconde, et les résultats de la discussion intéressent non seulement les navigateurs et les explorateurs, mais les astronomes eux-mêmes.

Cette discussion est donnée *in extenso* dans un rapport spécial, inséré au Bulletin astronomique de l'Observatoire de Paris, qui sera distribué incessamment.

Photographic Observations of Comet 1922 *b* (*Skjellerup*) *made with the* 30-*in. Reflector at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The following positions, for the Equinox of 1922·0, are based on the places of the comparison stars in the Astronomische Gesellschaft Catalogues. The addition of the corrections for parallax and aberration (which have *not* been applied) makes them comparable with positions computed from elements for the Equinox of 1922·0, the times being corrected for light-passage. The assumed values of log Δ are 9·4564, and 9·5435 on the two days, the light-times (*not* applied) being 142ˢ·6 and 174ˢ·3. The observer was Mr. Davidson.

G.M.T. 1922.	Comet's R.A.	Comet's N. Decl.	Corrections for Parallax. R.A.	Corrections for Parallax. Decl.	Corrections for Aberration. R.A.	Corrections for Aberration. Decl.
d h m s	h m s	° ′ ″	ˢ	″	ˢ	″
June 21 11 51 34·9	13 42 15·58	47 54 33·9	+1·68	+9·3	+0·89	+13·7
July 3 11 34 44·5	15 39 19·76	40 14 59·0	+0·88	+7·2	+1·25	+10·6
3 11 46 42·5	15 39 23·46	40 14 33·6	+0·94	+7·6	+1·25	+10·6

Royal Observatory, Greenwich:
1922 *July* 13.

Erratum.

P. 400. The diagram in Mr. Atkinson's paper, *Note on the Pressure in the Reversing Layer in Stars*, was unfortunately printed upside down. It should be as given below.